Paper Reference(s)

9801/01

Edexcel

Mathematics

Advanced Extension Award

Wednesday 25 June 2014 – Morning

Time: 3 hours

Materials required for examination	**Items included with question papers**
Answer book (AB16)	Nil
Graph paper (ASG2)	
Mathematical Formulae (Pink)	

Candidates may NOT use a calculator in answering this paper.

Instructions to Candidates

In the boxes on the answer book provided, write the name of the examining body (Edexcel), your centre number, candidate number, the paper title (Mathematics), the paper reference (9801), your surname, initials and signature.
Check that you have the correct question paper.
Answers should be given in as simple a form as possible. e.g. $\frac{2\pi}{3}$, $\sqrt{6}$, $3\sqrt{2}$.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 7 questions in this question paper.
The total mark for this paper is 100, of which 7 marks are for style, clarity and presentation.
There are 8 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. The function f is given by

$$f(x) = \ln(2x - 5), \qquad x > 2.5$$

(a) Find $f^{-1}(x)$.

(2)

The function g has domain $x > 2$ and

$$fg(x) = \ln\left(\frac{x+10}{x-2}\right), \qquad x > 2$$

(b) Find $g(x)$ and simplify your answer.

(3)

(Total 5 marks)

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2. Given that

$$3\sin^2 x + 2\sin x = 6\cos x + 9\sin x\cos x$$

and that $-90° < x < 90°$,

find the possible values of $\tan x$.

(Total 6 marks)

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3. (a) On separate diagrams sketch the curves with the following equations. On each sketch you should mark the coordinates of the points where the curve crosses the coordinate axes.

(i) $y = x^2 - 2x - 3$

(ii) $y = x^2 - 2|x| - 3$

(iii) $y = x^2 - x - |x| - 3$

(7)

(b) Solve the equation

$$x^2 - x - |x| - 3 = x + |x|$$

(4)

(Total 11 marks)

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4. Given that

$$(1+x)^n = 1 + \sum_{r=1}^{\infty} \frac{n(n-1)\dots(n-r+1)}{1\times 2\times\dots\times r} x^r \qquad (|x|<1,\ x\in\mathbb{R},\ n\in\mathbb{R})$$

(a) show that

$$(1-x)^{-\frac{1}{2}} = \sum_{r=0}^{\infty} \binom{2r}{r}\left(\frac{x}{4}\right)^r$$

(5)

(b) show that $(9-4x^2)^{-\frac{1}{2}}$ can be written in the form $\displaystyle\sum_{r=0}^{\infty}\binom{2r}{r}\frac{x^{2r}}{3^q}$ and give q in terms of r.

(3)

(c) Find $\displaystyle\sum_{r=1}^{\infty}\binom{2r}{r}\times\frac{2r}{9}\times\left(\frac{x}{3}\right)^{2r-1}$

(3)

(d) Hence find the exact value of

$$\sum_{r=1}^{\infty}\binom{2r}{r}\times\frac{2r\sqrt{5}}{9}\times\frac{1}{5^r}$$

giving your answer as a rational number.

(2)

(Total 13 marks)

5. The square-based pyramid P has vertices A, B, C, D and E. The position vectors of A, B, C and D are **a**, **b**, **c** and **d** respectively where

$$\mathbf{a} = \begin{pmatrix} -2 \\ 3 \\ -1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 5 \\ 8 \\ -6 \end{pmatrix}, \quad \mathbf{c} = \begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix}, \quad \mathbf{d} = \begin{pmatrix} 6 \\ 1 \\ 1 \end{pmatrix}$$

(a) Find the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$, $\overrightarrow{BC}$, $\overrightarrow{BD}$ and $\overrightarrow{CD}$.

(3)

(b) Find

(i) the length of a side of the square base of P,

(ii) the cosine of the angle between one of the slanting edges of P and its base,

(iii) the height of P,

(iv) the position vector of E.

(9)

A second pyramid, identical to P, is attached by its square base to the base of P to form an octahedron.

(c) Find the position vector of the other vertex of this octahedron.

(3)

(Total 15 marks)

6. (i) A curve with equation $y = \mathrm{f}(x)$ has $\mathrm{f}(x) \geqslant 0$ for $x \geqslant a$ and

$$A = \int_a^b \mathrm{f}(x)\,\mathrm{d}x \quad \text{and} \quad V = \pi\int_a^b [\mathrm{f}(x)]^2\,\mathrm{d}x$$

where a and b are constants with $b > a$.

Use integration by substitution to show that for the positive constants r and h

$$\pi\int_{a+h}^{b+h} [r + \mathrm{f}(x - h)]^2\,\mathrm{d}x = \pi r^2\,(b - a) + 2\pi r A + V$$

(3)

(ii)

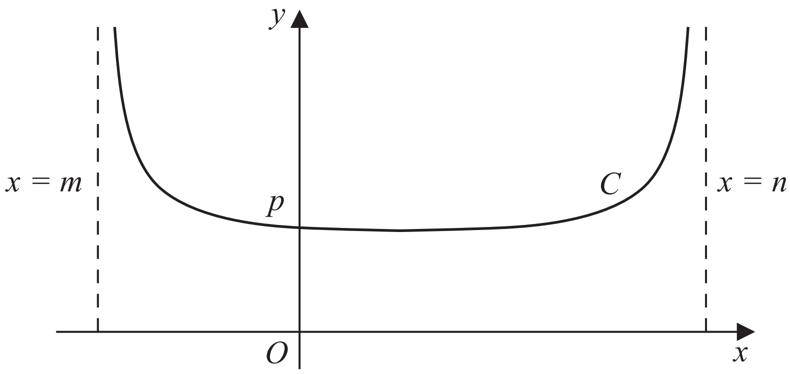

Figure 1

Figure 1 shows part of the curve C with equation $y = 4 + \dfrac{2}{\sqrt{3}\cos x + \sin x}$

This curve has asymptotes $x = m$ and $x = n$ and crosses the y-axis at $(0, p)$.

(a) Find the value of p, the value of m and the value of n.

(4)

(b) Show that the equation of C can be written in the form $y = r + \mathrm{f}(x - h)$ and specify the function f and the constants r and h.

(4)

The region bounded by C, the x-axis and the lines $x = \dfrac{\pi}{6}$ and $x = \dfrac{\pi}{3}$ is rotated through 2π radians about the x-axis.

(c) Find the volume of the solid formed.

(9)

(Total 20 marks)

7.

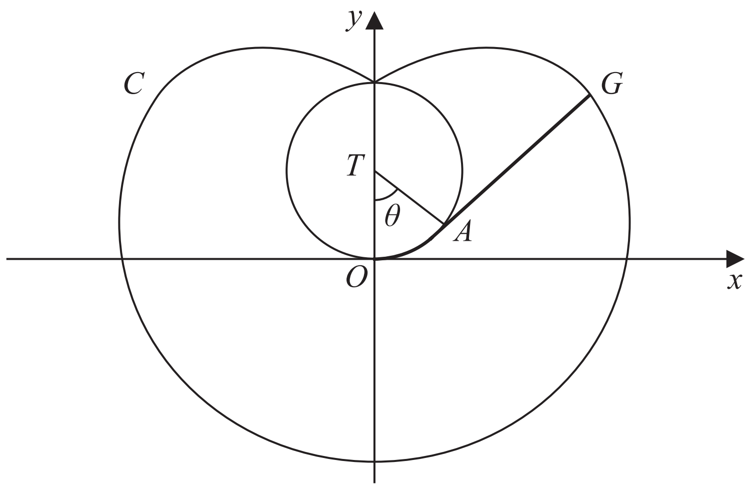

Figure 2

A circular tower stands in a large horizontal field of grass. A goat is attached to one end of a string and the other end of the string is attached to the fixed point O at the base of the tower. Taking the point O as the origin (0, 0), the centre of the base of the tower is at the point $T(0, 1)$. The radius of the base of the tower is 1. The string has length π and you may ignore the size of the goat. The curve C represents the edge of the region that the goat can reach as shown in Figure 2.

(a) Write down the equation of C for $y < 0$.

(1)

When the goat is at the point $G(x, y)$, with $x > 0$ and $y > 0$, as shown in Figure 2, the string lies along OAG where OA is an arc of the circle with angle $OTA = \theta$ radians and AG is a tangent to the circle at A.

(b) With the aid of a suitable diagram show that

$$x = \sin\theta + (\pi - \theta)\cos\theta$$

$$y = 1 - \cos\theta + (\pi - \theta)\sin\theta$$

(5)

(c) By considering $\int y \frac{\mathrm{d}x}{\mathrm{d}\theta}\,\mathrm{d}\theta$, show that the area between C, the positive x-axis and the positive y-axis can be expressed in the form

$$\int_0^{\pi} u \sin u \,\mathrm{d}u + \int_0^{\pi} u^2 \sin^2 u \,\mathrm{d}u + \int_0^{\pi} u \sin u \cos u \,\mathrm{d}u$$

(5)

(d) Show that $\int_0^{\pi} u^2 \sin^2 u \,\mathrm{d}u = \frac{\pi^3}{6} + \int_0^{\pi} u \sin u \cos u \,\mathrm{d}u$

(4)

(e) Hence find the area of grass that can be reached by the goat.

(8)

(Total 23 marks)

FOR STYLE, CLARITY AND PRESENTATION: 7 MARKS

TOTAL FOR PAPER: 100 MARKS

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