

C1 Lessons: Independent Study 2016

This booklet contains questions for you to do to support your work in lessons. The answers are included and you are expected to check your work and follow up any problems.

You can get help and extra questions from other students in M6, from your teachers and by borrowing the blue Higher GCSE book by Rayner or the red Understanding Pure Maths book by Sadler and Thorning.

Use this table to help you keep track of your work by filling in the dates the work is done and commenting on your confidence with it. Use the pages at the back of the booklet to record the independent study that you do.

Topic	Covered in lesson	Questions in booklet completed
Quadratics-factorising		
Quadratics-completing the square		
Coordinate Geometry		
Differentiation 1		
Differentiation 2		
Arithmetic Series		
Sequences		
Indices		
Integration		
Sketching graphs		
Simultaneous Equations		

In C1 you are given some formulae in the exam:

You probably won't use these: Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

You will definitely use these: Arithmetic series $u_n = a + (n - 1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n - 1)d]$$

A Quadratics Questions

1) Expand each of the following: a) $(x+1)(x+8)$ b) $(x+2)(x+9)$ c) $(x-2)(x-1)$
d) $(x-3)(x-12)$ e) $(x-12)(x+12)$ f) $(x+2)(x-11)$

2) Factorise, and solve, the following quadratics:

a) $x^2 + 3x + 2 = 0$ b) $x^2 + 5x + 6 = 0$ c) $x^2 - 7x + 12 = 0$
d) $x^2 + 3x - 10 = 0$ e) $x^2 - 3x - 28 = 0$ f) $x^2 - 20x + 51 = 0$

3) Factorise, solve, and then sketch, the following quadratics. Clearly label where the graph crosses axes.

a) $y = x^2 + 4x + 3$ b) $y = x^2 + 8x + 15$ c) $y = x^2 + x - 6$
d) $y = x^2 - 6x + 8$ e) $y = x^2 - x - 20$ f) $y = x^2 - 16$

3) Expand each of the following: a) $(2x+1)(x+8)$ b) $(3x+2)(2x+1)$ c) $(3x-2)(2x-1)$
d) $(5x-3)(x+8)$ e) $(3x-2)(3x+2)$ f) $(7x+2)(9x-11)$

4) Factorise, and solve, the following quadratics:

a) $2x^2 + 5x + 3 = 0$ b) $3x^2 + 8x + 4 = 0$ c) $2x^2 + x - 21 = 0$
d) $3x^2 - 11x + 10 = 0$ e) $6x^2 + 7x + 2 = 0$ f) $8x^2 - 10x - 3 = 0$

B Quadratics-Completing the Square Questions

1) Complete the square for each of, the following quadratics:

a) $x^2 + 4x$ b) $x^2 - 10x$ c) $x^2 + 2x + 10$ d) $x^2 - 8x + 16$ e) $x^2 + 3x$ f) $x^2 - x - 1$

2) Complete the square, and then sketch each of the following quadratics.

a) $y = x^2 + 2x - 6$ b) $y = x^2 - 6x + 8$ c) $y = x^2 - 3x$ d) $y = x^2 + x - 1$

3) Complete the square, and then solve each of the following quadratics (leaving your answers in surd form where necessary).

a) $x^2 + 2x - 2 = 0$ d) $x^2 - 12x + 8 = 0$ c) $x^2 - 5x = 0$ d) $x^2 + x - 13 = 0$

4) Complete the square, sketch, determine the co-ordinates of the vertex, and the equation of the line of symmetry for each of the following quadratics.

a) $y = x^2 + 6x + 5$ b) $y = x^2 + 7x + 12$ c) $y = x^2 + 4x - 1$

5) Complete the square for each of the following quadratics.

a) $y = 2x^2 + 4x$ b) $y = 4x^2 - 8x + 12$ c) $y = 2x^2 + 2x - 6$
d) $y = 2x^2 - 2x + 1$ e) $y = 3x^2 - 3x + 1$ f) $y = 2x^2 + x$

Quadratics - Answers

1. a) $x^2 + 9x + 8$ b) $x^2 + 11x + 18$ c) $x^2 - 3x + 2$ d) $x^2 - 15x + 36$ e) $x^2 - 144$ f) $x^2 - 9x - 22$

2. a) $x = -1, x = -2$ b) $x = -2, x = -3$ c) $x = 3, x = 4$ d) $x = -5, x = 2$ e) $x = -4, x = 7$
f) $x = 3, x = 17$

3. a) $x = -3, x = -1$ when $x = 0, y = 3$ b) $x = -3, x = -5$ when $x = 0, y = 15$

c) $x = 2, x = -3$ when $x = 0, y = -6$ d) $x = 2, x = 4$ when $x = 0, y = 8$

e) $x = -4, x = 5$ when $x = 0, y = -20$ f) $x = 4, x = -4$ when $x = 0, y = -16$

3 a) $2x^2 + 17x + 8$ b) $6x^2 + 7x + 2$ c) $6x^2 - 7x + 2$ d) $5x^2 + 37x - 24$ e) $9x^2 - 4$ f) $63x^2 - 59x - 22$

4. a) $(2x + 3)(x + 1) = 0$
 $x = -\frac{3}{2}, x = -1$ b) $(3x + 2)(x + 2) = 0$
 $x = -\frac{2}{3}, x = -2$ c) $(2x + 7)(x - 3) = 0$
 $x = -\frac{7}{2}, x = 3$ d) $(3x - 5)(x - 2) = 0$
 $x = \frac{5}{3}, x = 2$

e) $(3x + 2)(2x + 1) = 0$
 $x = -\frac{2}{3}, x = -\frac{1}{2}$ f) $(4x + 1)(2x - 3) = 0$
 $x = -\frac{1}{4}, x = \frac{3}{2}$

Completing the Square-Answers

1. a) $(x + 2)^2 - 4$ b) $(x - 5)^2 - 25$ c) $(x + 1)^2 + 9$ d) $(x - 4)^2$ e) $(x + \frac{3}{2})^2 - \frac{9}{4}$ f) $(x - \frac{1}{2})^2 - \frac{5}{4}$

2. a) $y = (x + 1)^2 - 7$ vertex $(-1, -7)$ crosses y-axis at $(0, -6)$

b) $y = (x - 3)^2 - 1$ vertex $(3, -1)$ crosses y-axis at $(0, 8)$

c) $y = (x - \frac{3}{2})^2 - \frac{9}{4}$ vertex $(\frac{3}{2}, -\frac{9}{4})$ crosses y-axis at $(0, 0)$

d) $y = (x + \frac{1}{2})^2 - \frac{5}{4}$ vertex $(-\frac{1}{2}, -\frac{5}{4})$ crosses y-axis at $(0, -1)$

3) a) $x = -1 \pm \sqrt{3}$ b) $x = 6 \pm 2\sqrt{7}$ c) $x = 5$ d) $x = \frac{-1 \pm \sqrt{53}}{2}$

4) a) $(-3, -4)$ $x = -3$ b) $(-3.5, -0.25)$ $x = -3.5$ c) $(-2, -5)$ $x = -2$

5) a) $y = 2(x + 1)^2 - 2$ b) $y = 4(x - 1)^2 + 8$ c) $y = 2(x + \frac{1}{2})^2 - 6\frac{1}{2}$

d) $y = 2(x - \frac{1}{2})^2 + \frac{1}{2}$ e) $y = 3(x - \frac{1}{2})^2 + \frac{1}{4}$ f) $y = 2(x + \frac{1}{4})^2 - \frac{1}{8}$

Coordinate Geometry Answers

1. a) 5 b) $\frac{4}{3}$ c) $-\frac{3}{4}$ d) (3.5, 5)

2. a) $3\sqrt{2}$ b) 1 c) -1 d) (0.5, 3.5)

3. a) gradient = 4, y-intercept = 1 b) gradient = 3, y-intercept = -2

 c) gradient = 2, y-intercept = $\frac{1}{2}$ d) gradient = 2, y-intercept = 5

 e) gradient = $\frac{1}{3}$, y-intercept = -2

4. a) $y = 3x - 3$ b) $3x - 2y + 7 = 0$ c) $x - 2y - 1 = 0$

 d) $2x + y = 0$ e) $x + 2y - 11 = 0$

Differentiation 1 Answers

A)

1) $\frac{dy}{dx} = 20x^3 + 2$

2) $\frac{dy}{dx} = 24x^3 - 6x + 2$

3) $\frac{dy}{dx} = -7x^{-2}$

4) $\frac{dy}{dx} = -4x^{-3}$

5) $f'(x) = 2/\sqrt{x}$

6) $f'(x) = 5/\sqrt{x}$

7) $f'(x) = -\frac{12}{x^2}$

8) $f'(x) = -\frac{5}{x^2}$

B) 1) $y = 21$ 2) $y = 87$ 3) $y = 8$ 4) $y = \frac{5}{4}$

C) 1) 68 2) 14 3) -3 4) $8\frac{1}{2}$

D) 1) $x = 3, y = -8$

 2) $x = 1, y = 4$

 3) $x = -1, y = 3$

 4) $x = -3, y = 5$ and $x = 1, y = -11$

E Differentiation 2 Questions

- A
- 1) Find the equation of the **tangent** to the curve $y = 5 - 2x^2$ at $(-1, 3)$ (Notice that you are given the value of y and don't need to find it). Give your answer in the form $y = mx + c$.
 - 2) Find the equation of the **tangent** to the curve $y = x^2 - 5x - 4$ at the point where $x = 5$. Give your answer in the form $y = mx + c$.
 - 3) Find the equation of the **normal** to the curve $y = 4 + x - 2x^2$ at the point where $x = 1$. Give your answer in the form $ax + by = c$.
- B
- Find $\frac{d^2y}{dx^2}$
- 1) $y = 6x^5 + 7x$
 - 2) $y = 4x^2 - 5x^4 + 2$
 - 3) $y = 6x^{-3}$
- C
- Find $f''(x)$
- 1) $f(x) = 2x^7 + 3x$
 - 2) $f(x) = \frac{12}{x}$
 - 3) $f(x) = 6\sqrt{x}$

F Arithmetic Series Questions

- 1) For the arithmetic sequence 3, 8, 13, 18, 23,... identify/evaluate:
 - a. The first term, a .
 - b. The common difference, d .
 - c. The 17th term.
 - d. The sum of the first 20 terms.
- 2) The first term of an arithmetic series is 5. The 12th term is 49. Work out the value of the common difference. The work out the sum of the first 15 terms.
- 3) The second term of an arithmetic series is 7 and the eighth term is 25. Write 2 equations for a and d , and solve them simultaneously.
- 4) Write out the terms of the series $\sum_{r=1}^8 r$
- 5) Calculate the sum of the first 20 natural numbers.

G Sequences Questions

- 1) Given $u_n = 3n - 2$, work out the first 4 terms of the sequence.
- 2) Given $u_{n+1} = 2u_n + 1$ and $u_1 = 2$, work out the values of u_2, u_3, u_4 .
- 3) Suggest a recurrence relation for the sequence 4, 7, 10, 13,...
- 4) A sequence has the formula $u_n = an + b$. Given that $u_2 = 14$ and $u_5 = 23$, work out the values of a and b .

Differentiation 2 Answers

- A
- 1) $y = 4x + 7$
 - 2) $y = 5x - 29$
 - 3) $x - 3y + 8 = 0$

B Find $\frac{d^2y}{dx^2}$

1) $\frac{d^2y}{dx^2} = 120x^3$

2) $\frac{d^2y}{dx^2} = 8 - 60x^2$

3) $\frac{d^2y}{dx^2} = 72x^{-5}$

C Find $f''(x)$

1) $f''(x) = 84x^5$

2) $f''(x) = \frac{24}{x^3}$

3) $f''(x) = -\frac{3}{2}x^{-3/2} = -\frac{3}{2x\sqrt{x}}$

Arithmetic series Answers

- 1) a. 3 b. 5 c. 83 d. 910
- 2) $d=4$, sum = 495
- 3) $a + d = 7$, $a + 7d = 25$. $a = 4$, $d = 3$.
- 4) $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8$
- 5) 210

Sequences Answers

1) 1, 4, 7, 10, 13

2) 5, 11, 23

3) $u_{n+1} = u_n + 3$, $u_1 = 4$.

4) $a = 3$, $b = 8$

H Indices Questions

1) Simplify each of the following:

a) $a^3 \times a^2$

b) $a^4 \div a^2$

c) $a^3 \times a^{-1}$

d) $a^{14} \div a^{-2}$

e) $a^{-3} \times a^{-2}$

f) $a^{-4} \div a^{-2}$

g) $a^{\frac{1}{2}} \times a^{-\frac{1}{4}}$

h) $a^{\frac{1}{4}} \div a^{\frac{1}{3}}$

2) Simplify each of the following:

a) $(a^3)^2$

b) $(a^{13})^3$

c) $(a^{-3})^2$

d) $(a^{-3})^{-2}$

e) $(a^3)^{\frac{1}{3}}$

f) $(a^{\frac{3}{2}})^{\frac{2}{3}}$

g) $(a^{-\frac{3}{2}})^{\frac{2}{3}}$

h) $(a^{-\frac{1}{4}})^{-\frac{2}{9}}$

3) Simplify each of the following:

a) $2a^3 \times 5a^5$

b) $9a^8 \div 3a^2$

c) $9a^1 \times \frac{1}{3}a^{-1}$

d) $2a^{-2} \div 4a^{-2}$

e) $(2a^4)^2$

f) $(3a)^3$

g) $(5a^{-13})^2$

h) $(2a^{-3})^{-2}$

I Integration Questions

1) Integrate the following functions with respect to x. Use clear notation.

a) $f(x) = 5x^2 + 3$ b) $f(x) = 2x - \sqrt{x}$ c) $f(x) = \frac{3}{x^2}$ d) $f(x) = \frac{1}{3x^2}$

2) $\int 3x^2 + 3x - 7dx$

3) $\int 3\sqrt{x} dx$

4) Find y in terms of x given that

a) $\frac{dy}{dx} = 3x^2 - 4x + 1$

b) $\frac{dy}{dx} = \frac{1}{5}x - x^{1/3}$

5) a) $\int (x + 3)(x - 4)dx$

b) $\int x^3(4 - 5x^2)dx$

c) $\int \frac{3x^4 - 5x}{x^3}dx$

Indices Answers

1. a) a^5 b) a^2 c) a^2 d) a^{16} e) a^{-5} f) a^{-2} g) $a^{\frac{1}{4}}$ h) $a^{-\frac{1}{12}}$

2. a) a^6 b) a^{39} c) a^{-6} d) a^6 e) a f) a g) $a^{-\frac{1}{3}}$ h) $a^{\frac{5}{6}}$

3. a) $10a^8$ b) $3a^6$ c) 3 d) $\frac{1}{2}a^{-4}$ e) $4a^8$ f) $27a^3$ g) $25a^{-26}$ h) $\frac{1}{4}a^6$

Integration Answers

1) a) $\frac{5}{3}x^3 + 3x + c$ b) $x^2 - \frac{2}{3}x^{3/2} + c$ c) $-\frac{3}{x} + c$ d) $-\frac{1}{3x} + c$

2) $x^3 + \frac{3}{2}x^2 - 7x + c$ 3) $2x^{3/2} + c$

4) a) $y = 3x^3 - 2x^2 + x + c$ b) $y = \frac{1}{10}x^2 - \frac{3}{4}x^{4/3} + c$

5) a) $\frac{1}{3}x^3 - \frac{1}{2}x^2 - 12x + c$ b) $x^4 - \frac{5}{6}x^6 + c$ c) $\frac{3}{2}x^2 + 5x^{-1} + c$

J Simultaneous Equations Questions

Solve the simultaneous equations:

1) $y = x^2 - 2x$ and $y = x + 4$

2) $y = x^2 - 2x + 7$ and $y = 7x - 8$

3) $y = x^2 - 3x + 7$ and $5x - y = 8$

4) $y = 2x^2$ and $9x - y = 4$

5) Verify that the two curves: $x^2 + y^2 = 20$ and $y = x - 2$ intersect at $(4, 2)$ and $(-2, -4)$.

6) Solve the simultaneous equations $xy = 8$ and $y = x + 10$

7) Explain why there are no solutions to the simultaneous equations: $x^2 + y^2 = 1$ and $y = x + 10$

Simultaneous Equations Answers

1. $x^2 - 3x - 4 = 0, x = -1, y = 3$ and $x = 4, y = 8$

2. $x^2 - 9x + 15 = 0$

$$x = \frac{9 \pm \sqrt{9^2 - (4 \times 15)}}{2}$$

$$y = \frac{47 + 7\sqrt{21}}{2}$$

$$x = \frac{9 \pm \sqrt{21}}{2}$$

$$y = \frac{47 - 7\sqrt{21}}{2}$$

3. $x^2 - 8x + 15 = 0$

$x = 3$

$y = 7$

$x = 5$

$y = 17$

4. $2x^2 - 9x + 4 = 0$

$x = \frac{1}{2}$

$y = \frac{1}{2}$

$x = 4$

$y = 32$

5. $2x^2 - 4x - 16 = 0 \Rightarrow x^2 - 2x - 8 = 0$

$x = 4$

$y = 2$

$x = -2$

$y = -4$

6. $x^2 + 2x - 8 = 0$

$x = 2$

$y = 4$

$x = -4$

$y = -2$

7. $2x^2 + 20x + 99 = 0$ Discriminant = $20^2 - 4 \times 2 \times 99 = -392$

A negative discriminant means that there are no solutions to the quadratic, hence no solution to the simultaneous equations.