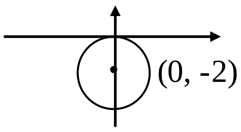
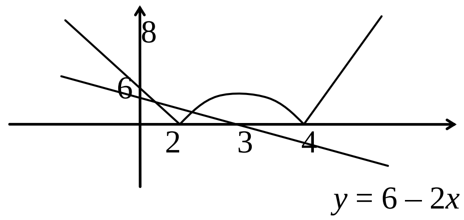
Further Pure Mathematics FP2 (6668)

Practice paper B mark scheme

Question number	Scheme	Marks
1. (a)	$\dfrac{r^2-(r-1)^2}{r^2(r-1)^2} = \dfrac{2r-1}{r^2(r-1)^2}$	M1 A1 (2)
(b)	$\displaystyle\sum_{r=2}^{n}\frac{2r-1}{r^2(r-1)^2} = \sum_{r=2}^{n}\frac{1}{(r-1)^2}-\frac{1}{r^2}$	M1
	$= \dfrac{1}{1^2}-\dfrac{1}{2^2}+\dfrac{1}{2^2}-\dfrac{1}{3^2}+\ldots+\dfrac{1}{(n-1)^2}-\dfrac{1}{n^2}$	M1
	$= 1 - \dfrac{1}{n^2}$ (✱)	A1 cso (3)
		(5 marks)
2. (a) (i)	$\lvert x + (y-2)\text{i}\rvert = 2\lvert x + (y+\text{i})\rvert$	M1
	$\therefore x^2 + (y-2)^2 = 4(x^2 + (y+1)^2)$	
	so $3x^2 + 3y^2 + 12y = 0$ any correct form; 3 terms; isw	A1 (2)
(ii)	 Sketch circle	B1
	Centre $(0, -2)$	B1
	$r = 2$ or touches axis	B1 (3)
(b)	$w = 3(z - 7 + 11\text{i})$	B1
	$= 3z - 21 + 33\text{i}$	B1 (2)
		(7 marks)
3.	$\dfrac{\mathrm{d}y}{\mathrm{d}x}+\dfrac{2}{1+x}y = \dfrac{1}{x(x+1)}$ Attempt $y' + Py = Q$ form	M1
	I.F. $= \mathrm{e}^{\int\frac{2}{1+x}\mathrm{d}x} = \mathrm{e}^{2\ln(1+x)},\ = (1+x)^2$	M1, A1
	$\therefore\ y(1+x)^2 = \displaystyle\int\left(\frac{x+1}{x}\right)\mathrm{d}x$ or $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(y(1+x)^2\right) = \dfrac{x+1}{x}$	M1ft
	i.e. $\left(y(1+x)^2 =\right)\ x + \ln x + (c)$	M1 A1
	$y = \dfrac{x+\ln x + c}{(1+x)^2}$	A1 cao (7)
		(7 marks)

Question number		Scheme	Marks
4.	(a)	$\mathrm{f}(x)=\cos 2x$, $\quad \mathrm{f}\left(\frac{\pi}{4}\right)=0$	
		$\mathrm{f}'(x)=-2\sin 2x$, $\quad \mathrm{f}'\left(\frac{\pi}{4}\right)=-2$	M1
		$\mathrm{f}''(x)=-4\cos 2x$, $\quad \mathrm{f}''\left(\frac{\pi}{4}\right)=0$	
		$\mathrm{f}'''(x)=8\sin 2x$, $\quad \mathrm{f}'''\left(\frac{\pi}{4}\right)=8$	A1
		$\mathrm{f}^{(\mathrm{iv})}(x)=16\cos 2x$, $\quad \mathrm{f}^{(\mathrm{iv})}\left(\frac{\pi}{4}\right)=0$	
		$\mathrm{f}^{(\mathrm{v})}(x)=-32\sin 2x$, $\quad \mathrm{f}^{(\mathrm{v})}\left(\frac{\pi}{4}\right)=-32$	A1
		$\cos 2x=\mathrm{f}(\frac{\pi}{4})+\mathrm{f}'(\frac{\pi}{4})(x-\frac{\pi}{4})+\frac{\mathrm{f}''(\frac{\pi}{4})}{2}(x-\frac{\pi}{4})^2+\frac{\mathrm{f}'''(\frac{\pi}{4})}{3!}(x-\frac{\pi}{4})^3+\ldots$ (three terms are sufficient to establish method)	M1
		$\cos 2x=-2\left(x-\frac{\pi}{4}\right)+\frac{4}{3}\left(x-\frac{\pi}{4}\right)^3-\frac{4}{15}\left(x-\frac{\pi}{4}\right)^5+\ldots$	A1 (5)
	(b)	Substitute $x=1$ $\quad \left(1-\frac{\pi}{4}\approx 0.21460\right)$	B1
		$\cos 2=-2\left(1-\frac{\pi}{4}\right)+\frac{4}{3}\left(1-\frac{\pi}{4}\right)^3-\frac{4}{15}\left(1-\frac{\pi}{4}\right)^5+\ldots$	
		≈ -0.416147	M1 A1 cao
			(3)
			(8 marks)
5.	(a)	$(\cos\theta+\mathrm{i}\sin\theta)^5=\cos 5\theta+\mathrm{i}\sin 5\theta$	M1
		$(\cos\theta+\mathrm{i}\sin\theta)^5=$	
		$\cos^5\theta+5\cos^4\theta(\mathrm{i}\sin\theta)+10\cos^3\theta(\mathrm{i}\sin\theta)^2$	
		$+10\cos^2\theta(\mathrm{i}\sin\theta)^3+5\cos\theta(\mathrm{i}\sin\theta)^4+(\mathrm{i}\sin\theta)^5$	M1 A1
		$\cos 5\theta=\cos^5\theta-10\cos^3\theta\sin^2\theta+5\cos\theta\sin^4\theta$	M1
		$=\cos^5\theta-10\cos^3\theta(1-\cos^2\theta)+$ $5\cos\theta(1-2\cos^2\theta+\cos^4\theta)$	M1
		$=16\cos^5\theta-20\cos^3\theta+5\cos\theta$ (✱)	A1 cso (6)
	(b)	$\cos 5\theta=-1$ (or 1, or 0)	M1
		$5\theta=(2n\pm 1)180°$ $\quad \Rightarrow \theta=(2n\pm 1)36°$	A1
		$x=\cos\theta=-1,\ -0.309,\ 0.809$	M1 A1 (4)
			(10 marks)

Question number	Scheme	Marks
6. (a)	$y = \lvert (x-2)(x-4) \rvert$ Line crosses axes	B1
	Curve shape	B1
	 Axes contacts:	
	6, 8	B1
	3, 2, 4	B1 (4)
(b)	$6 - 2x = (x-2)(x-4)$ and $-6 + 2x = (x-2)(x-4)$	M1 M1
	$x^2 - 4x + 2 = 0$ $x^2 - 8x + 14 = 0$ either	M1
	$x = \dfrac{4 \pm \sqrt{16-8}}{2}$ $x = \dfrac{8 \pm \sqrt{64-56}}{2}$	
	$= 2 - \sqrt{2}$ $= 4 - \sqrt{2}$	A1 A1 (5)
(c)	$2 - \sqrt{2} < x < 4 - \sqrt{2}$	M1 A1 (2)
		(11 marks)
7. (a)	$\dfrac{dy}{dx} = x\dfrac{dv}{dx} + v$, $\dfrac{d^2y}{dx^2} = x\dfrac{d^2v}{dx^2} + 2\dfrac{dv}{dx}$	M1 A1
	$\therefore\ x^2\left(x\dfrac{d^2v}{dx^2} + 2\dfrac{dv}{dx}\right) - 2x\left(x\dfrac{dv}{dx} + v\right) + (2 + 9x^2)vx = x^5$	M1
	$x^3\dfrac{d^2v}{dx^2} + 2x^2\dfrac{dv}{dx} - 2x^2\dfrac{dv}{dx} - 2vx + 2vx + 9vx^3 = x^5$	A1
	$\left[x^3\dfrac{d^2v}{dx^2} + 9vx^3 = x^5\right]$	
	Given result $\dfrac{d^2v}{dx^2} + 9v = x^2$ (✱)	A1 cso (5)
(b)	CF: $v = A\sin 3x + B\cos 3x$	M1 A1
	Appropriate form for PI: $v = \lambda x^2 + \mu$ (or $ax^2 + bx + c$)	M1
	Complete method to find λ and μ	M1
	$v = A\sin 3x + B\cos 3x + \dfrac{1}{9}x^2 - \dfrac{2}{81}$	M1 A1ft (6)
(c)	$\therefore\ y = Ax\sin 3x + Bx\sin 3x + \dfrac{1}{9}x^3 - \dfrac{2}{81}x$	B1ft (1)
		(12 marks)

Question number		Scheme	Marks
8.	(a) (i)	$r^2\sin^2\theta = a^2\cos 2\theta\sin^2\theta = a^2\left(1-2\sin^2\theta\right)\sin^2\theta$	B1 (1)
		$\left(= a^2\left(\sin^2\theta - 2\sin^4\theta\right)\right)$	
	(ii)	$\frac{\mathrm{d}}{\mathrm{d}\theta}\left(a^2\left(\sin^2\theta-2\sin^4\theta\right)\right) = a^2\left(2\sin\theta\cos\theta - 8\sin^3\theta\cos\theta\right)$, $= 0$	M1 A1, M1
		$2 = 8\sin^2\theta$ (Proceed to $a\sin^2\theta = b$)	M1
		$\sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$, $r = \frac{a}{\sqrt{2}}$ (✱)	A1 A1 cso
			(6)
	(b)	$\frac{a^2}{2}\int\cos 2\theta\,\mathrm{d}\theta = \frac{a^2}{4}\sin 2\theta$	M1 A1
		$\left[\ldots\right]_{\pi/6}^{\pi/4} = \frac{a^2}{4}\left[1-\frac{\sqrt{3}}{2}\right]$	M1 A1
		$\Delta = \frac{1}{2}\left(\frac{a}{\sqrt{2}}.\frac{1}{2}\right)\times\left(\frac{a}{\sqrt{2}}.\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}a^2}{16}$	M1 A1
		$R = \frac{\sqrt{3}a^2}{16} - \frac{a^2}{4}\left[1-\frac{\sqrt{3}}{2}\right] = \frac{a^2}{16}\left(3\sqrt{3}-4\right)$ (✱)	M1 A1 cso
			(8)
			(15 marks)