Paper Reference(s)

6668/01

Edexcel GCE

Further Pure Mathematics FP2

Advanced

Practice Paper B

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae	Answer Booklet

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 4 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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Turn over

1. (a) Express as a simplified single fraction $\dfrac{1}{(r-1)^2} - \dfrac{1}{r^2}$.

(2)

(b) Hence prove, by the method of differences, that

$$\sum_{r=2}^{n} \frac{2r-1}{r^2(r-1)^2} = 1 - \frac{1}{n^2} \ .$$

(3)

(Total 5 marks)

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2. (a) The point P represents a complex number z in an Argand diagram. Given that

$$|z - 2\mathrm{i}| = 2|z + \mathrm{i}|,$$

(i) find a cartesian equation for the locus of P, simplifying your answer.

(2)

(ii) sketch the locus of P.

(3)

(b) A transformation T from the z-plane to the w-plane is a translation $-7 + 11\mathrm{i}$ followed by an enlargement with centre the origin and scale factor 3.

Write down the transformation T in the form

$$w = az + b, \quad a, b \in \mathbb{C}.$$

(2)

(Total 7 marks)

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3. Find the general solution of the differential equation

$$(x + 1)\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = \frac{1}{x}, \qquad x > 0.$$

giving your answer in the form $y = \mathrm{f}(x)$.

(Total 7 marks)

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4. (a) Find the Taylor expansion of $\cos 2x$ in ascending powers of $\left(x - \dfrac{\pi}{4}\right)$ up to and including the term in $\left(x - \dfrac{\pi}{4}\right)^5$.

(5)

(b) Use your answer to (a) to obtain an estimate of $\cos 2$, giving your answer to 7 decimal places.

(3)

(Total 8 marks)

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5. (a) Use de Moivre's theorem to show that

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta.$$

(6)

(b) Hence find 3 distinct solutions of the equation $16x^5 - 20x^3 + 5x + 1 = 0$, giving your answers to 3 decimal places where appropriate.

(4)

(Total 10 marks)

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6. (a) Sketch, on the same axes, the graph of $y = |(x-2)(x-4)|$ and the line with equation $y = 6 - 2x$.

(4)

(b) Find the exact values of x for which $|(x-2)(x-4)| = 6 - 2x$.

(5)

(c) Hence solve the inequality $|(x-2)(x-4)| < 6 - 2x$.

(2)

(Total 11 marks)

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7. (a) Show that the transformation $y = xv$ transforms the equation

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + (2 + 9x^2)y = x^5, \qquad \text{I}$$

into the equation

$$\frac{d^2v}{dx^2} + 9v = x^2. \qquad \text{II}$$

(5)

(b) Solve the differential equation II to find v as a function of x.

(6)

(c) Hence state the general solution of the differential equation I.

(1)

(Total 12 marks)

8.

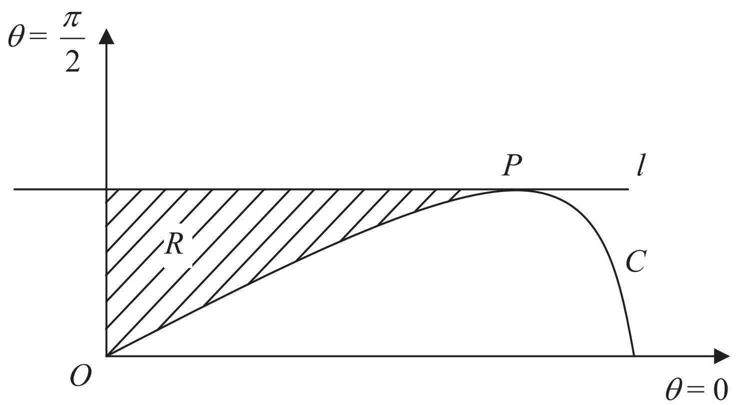

Figure 1

A curve C has polar equation $r^2 = a^2 \cos 2\theta$, $0 \leqslant \theta \leqslant \frac{\pi}{4}$. The line l is parallel to the initial line, and l is the tangent to C at the point P, as shown in Figure 1.

(a) (i) Show that, for any point on C, $r^2 \sin^2 \theta$ can be expressed in terms of $\sin \theta$ and a only.

(1)

(ii) Hence, using differentiation, show that the polar coordinates of P are $\left(\frac{a}{\sqrt{2}}, \frac{\pi}{6}\right)$.

(6)

The shaded region R, shown in Figure 1, is bounded by C, the line l and the half-line with equation $\theta = \frac{\pi}{2}$.

(b) Show that the area of R is $\frac{a^2}{16}(3\sqrt{3} - 4)$.

(8)

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

END