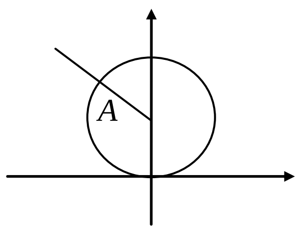
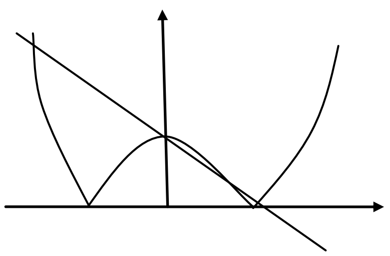
Further Pure Mathematics FP2 (6668)

Practice paper A mark scheme

Question number		Scheme	Marks
1.	(a)	$\cos\frac{\pi}{2} + \text{i}\sin\frac{\pi}{2}$	B1
		$\cos\frac{\pi}{10} + \text{i}\sin\frac{\pi}{10}$	B1
		$\cos\left(\frac{(4k+1)\pi}{10}\right) + \text{i}\sin\left(\frac{(4k+1)\pi}{10}\right),\ k = 2, 3, 4$ (or equivalent.)	M1
		$\cos\left(\frac{9\pi}{10}\right) + \text{i}\sin\left(\frac{9\pi}{10}\right),\ \cos\left(\frac{13\pi}{10}\right) + \text{i}\sin\left(\frac{13\pi}{10}\right),\ \cos\left(\frac{17\pi}{10}\right) + \text{i}\sin\left(\frac{17\pi}{10}\right)$	A2, 1, 0 (5)
		[Degrees : 18, 90, 162, 234, 306]	
			(5 marks)
2.	(a)	$\frac{2}{4r^2-1} = \frac{1}{2r-1} - \frac{1}{2r+1}$	B1
		$\sum_{r=1}^{n}\frac{2}{4r^2-1} = \frac{1}{1} - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \ldots + \frac{1}{2n-1} - \frac{1}{2n+1}$ Attempt method of differences.	M1
		$= 1 - \frac{1}{2n+1}$ (✱)	A1 cso (3)
	(b)	Sum $= \left(\frac{1}{2}\right)\left[\text{f}(20) - \text{f}(10)\right]$	M1
		$= \frac{1}{2}\left[1 - \frac{1}{41} - 1 + \frac{1}{21}\right] = \frac{10}{21\times 41}$ or $\frac{10}{861}$	A1 cso (2)
			(5 marks)

Question number		Scheme	Marks
3.	(a)	Use of $\frac{1}{2}\int r^2\,\mathrm{d}\theta$	B1
		Limits are $\frac{\pi}{8}$ and $\frac{\pi}{4}$	B1
		$16a^2\cos^2 2\theta = 8a^2\left(1+\cos 4\theta\right)$	M1
		$\int\left(1+\cos 4\theta\right)\mathrm{d}\theta = \theta + \dfrac{\sin 4\theta}{4}$	M1 A1
		$A = 4a^2\left[\theta + \dfrac{\sin 4\theta}{4}\right]_{\pi/8}^{\pi/4}$	
		$= a^2\left[4\left(\dfrac{\pi}{4}-\dfrac{\pi}{8}\right)+\left(0-1\right)\right]$	M1
		$= a^2\left(\dfrac{\pi}{2}-1\right) = \dfrac{1}{2}a^2\left(\pi-2\right)$ (✱)	A1 cso (7)
			(7 marks)
4.	(a)	 Circle	M1
		Correct circle [centre (0, 3) radius 3]	A1 (2)
	(b)	Drawing correct **half**-line passing as shown	B1
		Find either *x* or *y* coord. of *A*	M1 A1
		$z = -\dfrac{3\sqrt{2}}{2} + \left(3+\dfrac{3\sqrt{2}}{2}\right)\mathrm{i}$	A1 (4)
	(c)	$\lvert z-3\mathrm{i}\rvert = 3 \quad \rightarrow \quad \left\lvert \dfrac{2\mathrm{i}}{w} - 3\mathrm{i}\right\rvert = 3$	M1
		$\Rightarrow \dfrac{\lvert 2\mathrm{i}-3\mathrm{i}w\rvert}{\lvert w\rvert} = 3$	A1
		$\Rightarrow \left\lvert w-\dfrac{2}{3}\right\rvert = \lvert w\rvert$	M1 A1
		Line with equation $u = \frac{1}{3}$ $\quad (x = \frac{1}{3})$	A1 (5)

Question number	Scheme	Marks
	Alternatives for part (c):	
(c)	$w = \frac{2\text{i}}{x + \text{i}y} = \frac{2\text{i}(x - \text{i}y)}{x^2 + y^2} \Rightarrow u = \frac{2y}{x^2 + y^2},\ v = \frac{2x}{x^2 + y^2}$	M1 A1
	As $x^2 + y^2 - 6y = 0$, $u = \frac{1}{3}$	M1 A1 A1
(c)	$w = \frac{2\text{i}}{3\cos\theta + 3\text{i}(1 + \sin\theta)} = \frac{2\text{i}[\cos\theta - \text{i}(1 + \sin\theta)]}{3\{\cos^2\theta + (1 + \sin\theta)^2\}}$	M1 A1
	$= \frac{2}{3}\frac{(1 + \sin\theta) + \text{i}\cos\theta}{2 + 2\sin\theta} = \frac{1}{3} + \text{i}\frac{\cos\theta}{1 + \sin\theta}$	M1 A1
	so locus is line $u = \frac{1}{3}$	A1
		(11 marks)
5. (a)	Correct method for producing 2nd order differential equation	
	e.g. $\frac{\text{d}}{\text{d}x}\left\{(1 + 2x)\frac{\text{d}y}{\text{d}x}\right\} = \frac{\text{d}}{\text{d}x}\left\{x + 4y^2\right\}$ attempted	M1
	$(1 + 2x)\frac{\text{d}^2y}{\text{d}x^2} + 2\frac{\text{d}y}{\text{d}x} = 1 + 8y\frac{\text{d}y}{\text{d}x}$ seen, and conclusion (✱)	A1 (2)
(b)	Differentiating again w.r.t. x:	
	$(1 + 2x)\frac{\text{d}^3y}{\text{d}x^3} + 2\frac{\text{d}^2y}{\text{d}x^2} = 8y\frac{\text{d}^2y}{\text{d}x^2} + 8\left(\frac{\text{d}y}{\text{d}x}\right)^2 - 2\frac{\text{d}^2y}{\text{d}x^2}$ or equiv.	M1 A2, 1, 0 (3)
	[e.g. $(1 + 2x)\frac{\text{d}^3y}{\text{d}x^3} = 8\left(\frac{\text{d}y}{\text{d}x}\right)^2 + 2(4y - 1)\frac{\text{d}^2y}{\text{d}x^2}$]	
(c)	$\frac{\text{d}y}{\text{d}x}$ (at $x = 0$) = 1	B1
	Finding $\frac{\text{d}^2y}{\text{d}x^2}$ (at $x = 0$) (= 3)	M1
	Finding $\frac{\text{d}^3y}{\text{d}x^3}$, at $x = 0$; = 8	M1 A1ft
	$y = \frac{1}{2} + x + \frac{3}{2}x^2 + \frac{4}{3}x^3 + \ldots$	M1 A1 (6)
		(11 marks)

Question number	Scheme	Marks
6. (a)	Auxiliary equation: $m^2+2m+2=0 \rightarrow m=-1\pm \mathrm{i}$	M1
	C. F. is $y=e^{-t}(A\cos t+B\sin t)$	M1 A1
	P.I. is $y=\lambda e^{-t}$, with $y'=-\lambda e^{-t}$, and $y''=\lambda e^{-t}$	M1
	$\therefore (\lambda-2\lambda+2\lambda)e^{-t}=2e^{-t} \rightarrow \lambda=2$	A1
	$\therefore y=e^{-t}(A\cos t+B\sin t+2)$	B1 (6)
(b)	Puts 1 = A + 2 and solves to obtain $A=-1$	M1
	$y'=e^{-t}(-A\sin t+B\cos t)-e^{-t}(A\cos t+B\sin t+2)$	M1 A1ft
	Puts 1 = $B-A-2$ and uses value for A to obtain B	M1
	$B=2$	A1 cso
	$\therefore y=e^{-t}(2\sin t-\cos t+2)$	A1 cso (6)
		(12 marks)
7. (a)	$2x^2+x-6=6-3x$	M1
	Leading to $x^2+2x-6=0$	
	$(x+1)^2=7 \Rightarrow x=-1\pm\sqrt{7}$ surds required	M1 A1
	$-2x^2-x+6=6-3x$	M1
	Leading to $2x^2-2x=0 \Rightarrow x=0,1$	A1 A1 (6)
	(accept if parts (a) and (b) done in reverse order)	
(b)	 Curved shape	B1
	Line	B1
	At least 3 intersections	B1 (3)
(c)	Using all 4 CVs and getting all into inequalities	M1
	$x>\sqrt{7}-1$, $x<-\sqrt{7}-1$ both	A1ft
	$0<x<1$	A1 (3)
		(12 marks)

Question number	Scheme	Marks
8. (a)	$\int \frac{2}{120-t}\,\mathrm{d}t = -2\ln(120-t)$	B1
	$\mathrm{e}^{-2\ln(120-t)} = (120-t)^{-2}$	M1 A1
	$\frac{1}{(120-t)^2}\frac{\mathrm{d}S}{\mathrm{d}t} + \frac{2S}{(120-t)^3} = \frac{1}{4(120-t)^2}$	
	$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{S}{(120-t)^2}\right) = \frac{1}{4(120-t)^2}$ or integral equivalent	M1
	$\frac{S}{(120-t)^2} = \frac{1}{4(120-t)}\ (+C)$	M1 A1
	$(0, 6) \Rightarrow 6 = 30 + 120^2 C \Rightarrow C = -\frac{1}{600}$	M1
	$S = \frac{120-t}{4} - \frac{(120-t)^2}{600}$ accept C = awrt -0.0017	A1 (8)
(b)	$\frac{\mathrm{d}S}{\mathrm{d}t} = -\frac{1}{4} + \frac{2(120-t)}{600}$	M1
	$\frac{\mathrm{d}S}{\mathrm{d}t} = 0 \Rightarrow t = 45$	M1 A1
	Substituting, $S = 9\frac{3}{8}$ (kg)	A1 (4)
		(12 marks)