

Mark Scheme (Results)

June 2013

GCE Further Pure Mathematics FP2
(6668/01)
Original Paper

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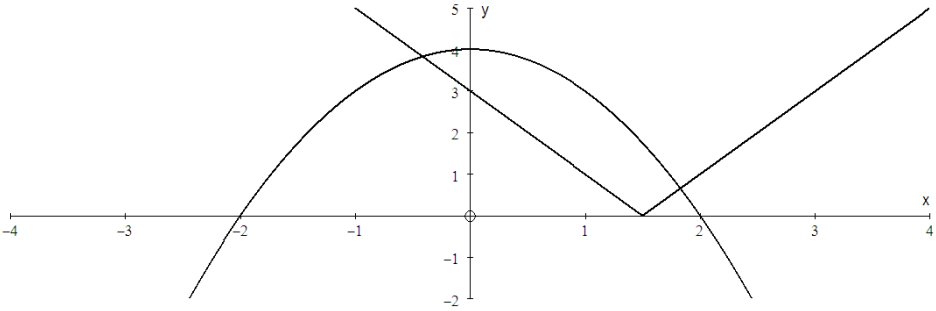
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

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Further Pure Mathematics FP2 6668
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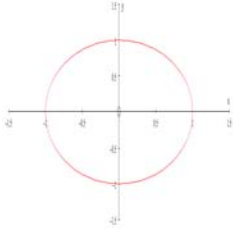
Question Number	Scheme	Marks
1	<p>(a) $y''' + xy'' + y' = -2\sin x$ so $y''' = -xy'' - y' - 2\sin x$</p> <p>(b) $y''' + 3 = 0$ so $y''' = -3$</p> <p>(c) Substitute to give $\frac{d^2y}{dx^2} = 2$</p> <p>Use Taylor Expansion to give $y = 1 + 3x + x^2 - \frac{x^3}{2} \dots$</p>	<p>M1, A1, A1 (3)</p> <p>B1 (1)</p> <p>B1</p> <p>M1 A1 (3)</p> <p>(7)</p>
2.	<p>(a)</p>  <p>(b) Put $4 - x^2 = 2x - 3$ or $4 - x^2 = -2x + 3$</p> <p>Solve $x^2 + 2x - 7 = 0$, to give $x = \frac{-2 + \sqrt{4 + 28}}{2} = -1 + 2\sqrt{2}$</p> <p>Solve $x^2 - 2x - 1 = 0$, to give $x = \frac{2 - \sqrt{4 + 4}}{2} = 1 - \sqrt{2}$</p> <p>So $1 - \sqrt{2} < x < 2\sqrt{2} - 1$</p>	<p>B1 (V shape) B1 (Parabola) B1 (positions correct)</p> <p>(3)</p> <p>M1</p> <p>M1 A1</p> <p>M1 A1</p> <p>B1</p> <p>(6) (9)</p>

Question Number	Scheme	Marks
3.	<p>(a) $f'(x) = \frac{k \cos kx}{1 + \sin kx}$</p> <p>(b)</p> $f''(x) = \frac{-(1 + \sin kx)k^2 \sin kx - k \cos kx(k \cos kx)}{(1 + \sin kx)^2}$ <p>so $f''(x) = \frac{-k^2 \sin kx - k^2(\sin^2 kx + \cos^2 kx)}{(1 + \sin kx)^2}$ and use $\sin^2 kx + \cos^2 kx = 1$</p> $f''(x) = \frac{-k^2(1 + \sin kx)}{(1 + \sin kx)^2} = \frac{-k^2}{1 + \sin kx} \quad *$ <p>(c) $f'''(x) = \frac{k^3 \cos kx}{(1 + \sin kx)^2}$</p> <p>Finds $f(0), f'(0), f''(0)$ and $f'''(0)$</p> <p>Uses MacLaurin Expansion to obtain $f(x) = 0 + kx - \frac{k^2}{2}x^2 + \frac{k^3}{6}x^3 \dots$</p>	<p>M1 A1 (2)</p> <p>M1</p> <p>M1</p> <p>A1cao (3)</p> <p>B1</p> <p>M1</p> <p>M1 A1 (4)</p>
	<p>Alternative method for (c)</p> <p>Uses $\ln(1 + y) = y - \frac{y^2}{2} + \frac{y^3}{3} - \dots$ with $y = \sin kx$</p> <p>Use $\sin kx = kx - \frac{k^3 x^3}{6} \dots$ in \ln expansion</p> <p>Obtains $0 + kx - \frac{k^2}{2}x^2 + \frac{k^3}{6}x^3 \dots$</p>	<p>B1</p> <p>M1</p> <p>M1 A1 (4)</p>

Question Number	Scheme	Marks
4.	$\text{IF} = \int \frac{1+x \cot x}{x} dx$ <p>Obtains $\ln x + \ln \sin x$</p> <p>{So IF = $x \sin x$}</p> $\frac{d}{dx}(\text{IF}y) = \text{IF} \times \frac{\sin x}{x}$ $yx \sin x = \int \sin^2 x dx = \int \frac{1 - \cos 2x}{2} dx = \frac{x}{2} - \frac{1}{4} \sin 2x (+c)$ <p>So $y = \frac{1}{2} \operatorname{cosec} x - \frac{1}{2x} \cos x + \frac{c}{x \sin x}$</p>	<p>B1</p> <p>M1 A1</p> <p>A1ft</p> <p>M1</p> <p>M1 A1</p> <p>M1 A1</p> <p style="text-align: right;">(9)</p>

Question Number	Scheme	Marks
5	<p>(a) $\frac{1}{r} - \frac{2}{(r+1)} + \frac{1}{(r+2)}$</p> <p>(b) LHS = $\frac{1}{1} - \frac{2}{2} + \frac{1}{3}$</p> <p style="padding-left: 40px;">$+ \frac{1}{2} - \frac{2}{3} + \frac{1}{4}$</p> <p style="padding-left: 40px;">$+ \frac{1}{3} - \frac{2}{4} + \frac{1}{5}$</p> <p style="padding-left: 40px;">.....</p> <p style="padding-left: 100px;">$+ \frac{1}{(n-2)} - \frac{2}{(n-1)} + \frac{1}{n}$</p> <p style="padding-left: 100px;">$+ \frac{1}{(n-1)} - \frac{2}{n} + \frac{1}{(n+1)}$</p> <p style="padding-left: 100px;">$+ \frac{1}{n} - \frac{2}{(n+1)} + \frac{1}{(n+2)}$</p> <p>$= 1 - \frac{1}{2} - \frac{1}{n+1} + \frac{1}{n+2} = \frac{n^2 + 3n + 2 - 2n - 4 + 2n + 2}{2(n+1)(n+2)}$</p> <p>$= \frac{n^2 + 3n}{2(n+1)(n+2)} = \frac{n(n+3)}{2(n+1)(n+2)} \quad *$</p>	<p>M1 A1 A1 (3)</p> <p>M1</p> <p>A1</p> <p>M1 dep</p> <p>A1 (4)</p>

Question Number	Scheme	Marks
6	<p>Modulus = 32</p> <p>Argument = $\arctan\left(-\frac{1}{\sqrt{3}}\right) = \frac{5\pi}{6}$</p> $z = 32^{\frac{1}{5}} \left(\cos\left(\frac{5\pi}{6}\right) + i \sin\left(\frac{5\pi}{6}\right) \right)^{\frac{1}{5}} = 2 \left(\cos\left(\frac{\pi}{6}\right) + i \sin\left(\frac{\pi}{6}\right) \right)$ <p>OR $2 \left(\cos\left(\frac{\pi}{6} + \frac{2n\pi}{5}\right) + i \sin\left(\frac{\pi}{6} + \frac{2n\pi}{5}\right) \right)$</p> <p>$= 2 \left(\cos\left(\frac{17\pi}{30}\right) + i \sin\left(\frac{17\pi}{30}\right) \right), 2 \left(\cos\left(\frac{29\pi}{30}\right) + i \sin\left(\frac{29\pi}{30}\right) \right),$</p> <p>$2 \left(\cos\left(\frac{-7\pi}{30}\right) + i \sin\left(\frac{-7\pi}{30}\right) \right), 2 \left(\cos\left(\frac{-19\pi}{30}\right) + i \sin\left(\frac{-19\pi}{30}\right) \right)$</p>	<p>B1</p> <p>M1A1</p> <p>M1 A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>(8)</p>
7(a)	<p>Differentiate twice and obtaining</p> $\frac{dy}{dx} = 2\lambda x e^{2x} + \lambda e^{2x} \quad \text{and} \quad \frac{d^2y}{dx^2} = 4\lambda x e^{2x} + 2\lambda e^{2x} + 2\lambda e^{2x}$ <p>Substitute to give $\lambda = \frac{3}{2}$</p>	<p>M1 A1</p> <p>M1 A1</p> <p>(4)</p>
(b)	<p>Complementary function is $y = Ae^{2x} + Be^{-2x}$</p> <p>So general solution is $y = Ae^{2x} + Be^{-2x} + \frac{3}{2}xe^{2x}$</p>	<p>M1 A1</p> <p>A1</p> <p>(3)</p> <p>(7)</p>

Question Number	Scheme	Marks
8.	<p>(a)</p>  <p>(b) $w(z-2i) = z+7i$ so $z(w-1) = 7i + 2iw$ and $z =$</p> $\frac{7i + 2iw}{w-1}$ <p>So $7i + 2iw = w-1$</p> <p>Using $w = u + iv$, $(-2v)^2 + (2u+7)^2 = (u-1)^2 + v^2$</p> <p>So $3u^2 + 3v^2 + 30u + 48 = 0$, which is a circle equation</p> <p>As $(u+5)^2 + v^2 = 3^2$</p> <p>So centre is -5 and radius is 3</p>	<p>B1</p> <p>(1)</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>(5)</p> <p>M1</p> <p>A1</p> <p>(2)</p> <p>(8)</p>

Question Number	Scheme	Marks
Q9 (a)	$2 - 2\sin\theta = 1 \rightarrow \sin\theta = 0.5 \therefore \theta = \frac{\pi}{6} \left(\text{or } \frac{5\pi}{6} \right),$ <p>Points are $(1, \frac{\pi}{6})$ and $(1, \frac{5\pi}{6})$</p>	M1 A1, A1 (3)
(b)	<p>Uses Area = $\frac{1}{2} \left[\int (2 - 2\sin\theta)^2 d\theta \right]$</p> $= \frac{1}{2} \left[\int (4 - 8\sin\theta + (2 - 2\cos 2\theta)) d\theta \right]$ $= \frac{1}{2} [(6\theta + 8\cos\theta - \sin 2\theta)] =$ <p>Uses limits $\frac{\pi}{2}$ and $\frac{\pi}{6}$ to give $\pi - \frac{7\sqrt{3}}{4}$ or $2\pi - \frac{7\sqrt{3}}{2}$</p> <p>Finds area of sector of circle $\frac{2\pi}{3}$</p> <p>So required area is $\frac{8\pi}{3} - \frac{7\sqrt{3}}{2}$</p>	M1 M1 A1 M1 A1 B1 M1 A1 (8)

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