edexcel
advancing learning, changing lives

Mark Scheme (Final)
Summer 2009

GCE

GCE Further Pure Mathematics FP2 (6668/01)

Edexcel Limited. Registered in England and Wales No. 4496750
Registered Office: One90 High Holborn, London WC1V 7BH

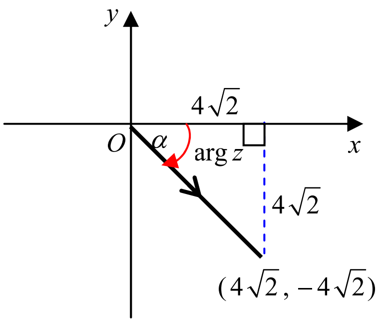
A PEARSON COMPANY

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

June 2009
6668 Further Pure Mathematics 2
Mark Scheme

Question Number	Scheme		Marks
1. (a)	$\frac{1}{r(r+2)} = \frac{1}{2r} - \frac{1}{2(r+2)}$	$\frac{1}{2r} - \frac{1}{2(r+2)}$	B1 **aef**
			(1)
(b)	$\sum_{r=1}^{n} \frac{4}{r(r+2)} = \sum_{r=1}^{n} \left(\frac{2}{r} - \frac{2}{r+2}\right)$		
	$= \left(\frac{2}{1} - \frac{2}{3}\right) + \left(\frac{2}{2} - \frac{2}{4}\right) + \;\ldots\ldots$ $\ldots\ldots\ldots\ldots + \left(\frac{2}{n-1} - \frac{2}{n+1}\right) + \left(\frac{2}{n} - \frac{2}{n+2}\right)$	List the first two terms and the last two terms	M1
	$= \frac{2}{1} + \frac{2}{2}; - \frac{2}{n+1} - \frac{2}{n+2}$	Includes the first two underlined terms and includes the final two underlined terms.	M1
		$\frac{2}{1} + \frac{2}{2} - \frac{2}{n+1} - \frac{2}{n+2}$	A1
	$= 3 - \frac{2}{n+1} - \frac{2}{n+2}$		
	$= \frac{3(n+1)(n+2) - 2(n+2) - 2(n+1)}{(n+1)(n+2)}$ $= \frac{3n^2 + 9n + 6 - 2n - 4 - 2n - 2}{(n+1)(n+2)}$	Attempt to combine to an at least 3 term fraction to a single fraction and an attempt to take out the brackets from their numerator.	M1
	$= \frac{3n^2 + 5n}{(n+1)(n+2)}$		
	$= \frac{n(3n+5)}{(n+1)(n+2)}$	Correct Result	A1 **cso** **AG**
			[5]
			6 marks

Question Number	Scheme		Marks
2. (a)	$z^3 = 4\sqrt{2} - 4\sqrt{2}\,\mathrm{i}\ ,\ \ -\pi < \theta \leq \pi$		
			
	$r = \sqrt{\left(4\sqrt{2}\right)^2 + \left(-4\sqrt{2}\right)^2} = \sqrt{32+32} = \sqrt{64} = 8$ $\theta = -\tan^{-1}\left(\frac{4\sqrt{2}}{4\sqrt{2}}\right) = -\frac{\pi}{4}$	A valid attempt to find the modulus and argument of $4\sqrt{2} - 4\sqrt{2}\,\mathrm{i}$.	M1
	$z^3 = 8\left(\cos\left(-\frac{\pi}{4}\right) + \mathrm{i}\sin\left(-\frac{\pi}{4}\right)\right)$		
	So, $z = (8)^{\frac{1}{3}}\left(\cos\left(\frac{-\frac{\pi}{4}}{3}\right) + \mathrm{i}\sin\left(\frac{-\frac{\pi}{4}}{3}\right)\right)$	Taking the cube root of the modulus and dividing the argument by 3.	M1
	$\Rightarrow\ z = 2\left(\cos\left(-\frac{\pi}{12}\right) + \mathrm{i}\sin\left(-\frac{\pi}{12}\right)\right)$	$2\left(\cos\left(-\frac{\pi}{12}\right) + \mathrm{i}\sin\left(-\frac{\pi}{12}\right)\right)$	A1
	Also, $z^3 = 8\left(\cos\left(\frac{7\pi}{4}\right) + \mathrm{i}\sin\left(\frac{7\pi}{4}\right)\right)$ or $z^3 = 8\left(\cos\left(-\frac{9\pi}{4}\right) + \mathrm{i}\sin\left(-\frac{9\pi}{4}\right)\right)$	Adding or subtracting 2π to the argument for z^3 in order to find other roots.	M1
	$\Rightarrow\ z = 2\left(\cos\frac{7\pi}{12} + \mathrm{i}\sin\frac{7\pi}{12}\right)$	Any one of the final two roots	A1
	and $z = 2\left(\cos\left(\frac{-3\pi}{4}\right) + \mathrm{i}\sin\left(\frac{-3\pi}{4}\right)\right)$	Both of the final two roots.	A1
			[6]
			6 marks

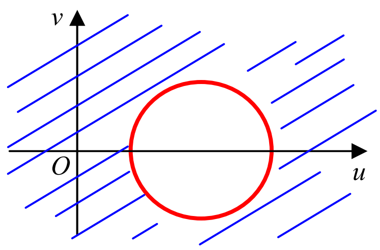
Special Case 1: Award SC: M1M1A1M1A0A0 for ALL three of $2\left(\cos\frac{\pi}{12} + \mathrm{i}\sin\frac{\pi}{12}\right)$, $2\left(\cos\frac{3\pi}{4} + \mathrm{i}\sin\frac{3\pi}{4}\right)$ and $2\left(\cos\left(\frac{-7\pi}{12}\right) + \mathrm{i}\sin\left(\frac{-7\pi}{12}\right)\right)$.

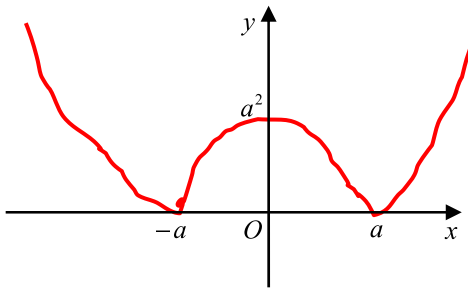
Special Case 2: If r is incorrect and candidate states the brackets ( ) correctly then give the first accuracy mark ONLY where this is applicable.

Question Number	Scheme		Marks
3.	$\sin x \dfrac{dy}{dx} - y\cos x = \sin 2x \sin x$		
	$\dfrac{dy}{dx} - \dfrac{y\cos x}{\sin x} = \dfrac{\sin 2x \sin x}{\sin x}$	An attempt to divide every term in the differential equation by $\sin x$. Can be implied. See appendix.	M1
	$\dfrac{dy}{dx} - \dfrac{y\cos x}{\sin x} = \sin 2x$		
	Integrating factor $= e^{\int -\frac{\cos x}{\sin x}dx} = e^{-\ln \sin x}$	$e^{\int \pm \frac{\cos x}{\sin x}(dx)}$ or $e^{\int \text{their P}(x)(dx)}$	dM1
		$e^{-\ln \sin x}$ or $e^{\ln \operatorname{cosec} x}$	A1 aef
	$= \dfrac{1}{\sin x}$	$\dfrac{1}{\sin x}$ or $(\sin x)^{-1}$ or $\operatorname{cosec} x$	A1 aef
	$\left(\dfrac{1}{\sin x}\right)\dfrac{dy}{dx} - \dfrac{y\cos x}{\sin^2 x} = \dfrac{\sin 2x}{\sin x}$		
	$\dfrac{d}{dx}\left(\dfrac{y}{\sin x}\right) = \sin 2x \times \dfrac{1}{\sin x}$	$\dfrac{d}{dx}(y \times \text{their I.F.}) = \sin 2x \times \text{their I.F.}$	M1
	$\dfrac{d}{dx}\left(\dfrac{y}{\sin x}\right) = 2\cos x$	$\dfrac{d}{dx}\left(\dfrac{y}{\sin x}\right) = 2\cos x$ or $\dfrac{y}{\sin x} = \int 2\cos x\,(dx)$	A1
	$\dfrac{y}{\sin x} = \int 2\cos x\, dx$		
	$\dfrac{y}{\sin x} = 2\sin x + K$	A credible attempt to integrate the RHS with/without $+K$	dddM1
	$y = 2\sin^2 x + K\sin x$	$y = 2\sin^2 x + K\sin x$	A1 **cao**
			[8]
			8 marks

Question Number	Scheme		Marks
4.	$A = \frac{1}{2}\int_0^{2\pi}(a+3\cos\theta)^2\,\mathrm{d}\theta$	Applies $\frac{1}{2}\int_0^{2\pi} r^2\,(\mathrm{d}\theta)$ with correct limits. Ignore $\mathrm{d}\theta$.	B1
	$(a+3\cos\theta)^2 = a^2 + 6a\cos\theta + 9\cos^2\theta$		
	$= \underline{a^2 + 6a\cos\theta + 9\left(\frac{1+\cos 2\theta}{2}\right)}$	$\cos^2\theta = \frac{\pm 1 \pm \cos 2\theta}{2}$	M1
		Correct underlined expression.	A1
	$A = \frac{1}{2}\int_0^{2\pi}\left(a^2 + 6a\cos\theta + \frac{9}{2} + \frac{9}{2}\cos 2\theta\right)\mathrm{d}\theta$		
	$= \left(\frac{1}{2}\right)\left[a^2\theta + 6a\sin\theta + \frac{9}{2}\theta + \frac{9}{4}\sin 2\theta\right]_0^{2\pi}$	Integrated expression with at least 3 out of 4 terms of the form $\pm A\theta \pm B\sin\theta \pm C\theta \pm D\sin 2\theta$. Ignore the $\frac{1}{2}$. Ignore limits.	M1*
		$a^2\theta + 6a\sin\theta +$ correct ft integration. Ignore the $\frac{1}{2}$. Ignore limits.	A1 ft
	$= \frac{1}{2}\left[\left(2\pi a^2 + 0 + 9\pi + 0\right) - (0)\right]$		
	$= \pi a^2 + \frac{9\pi}{2}$	$\pi a^2 + \frac{9\pi}{2}$	A1
	Hence, $\pi a^2 + \frac{9\pi}{2} = \frac{107}{2}\pi$	Integrated expression equal to $\frac{107}{2}\pi$.	dM1*
	$a^2 + \frac{9}{2} = \frac{107}{2}$		
	$a^2 = 49$		
	As $a > 0$, $a = 7$	$a = 7$	A1
			[8]
			8 marks

Question Number	Scheme		Marks
5.	$y = \sec^2 x = (\sec x)^2$		
(a)	$\frac{dy}{dx} = 2(\sec x)^1(\sec x \tan x) = 2\sec^2 x \tan x$	Either $2(\sec x)^1(\sec x \tan x)$ or $2\sec^2 x \tan x$	B1 aef
	Apply product rule: $\left\{ u = 2\sec^2 x \quad v = \tan x \quad \frac{du}{dx} = 4\sec^2 x \tan x \quad \frac{dv}{dx} = \sec^2 x \right\}$		
	$\frac{d^2y}{dx^2} = 4\sec^2 x \tan^2 x + 2\sec^4 x$	Two terms added with one of either $A\sec^2 x \tan^2 x$ or $B\sec^4 x$ in the correct form.	M1
		Correct differentiation	A1
	$= 4\sec^2 x(\sec^2 x - 1) + 2\sec^4 x$		
	Hence, $\frac{d^2y}{dx^2} = 6\sec^4 x - 4\sec^2 x$	Applies $\tan^2 x = \sec^2 x - 1$ leading to the correct result.	A1 **AG**
			[4]
(b)	$y_{\frac{\pi}{4}} = \left(\sqrt{2}\right)^2 = \underline{2}$, $\left(\frac{dy}{dx}\right)_{\frac{\pi}{4}} = 2\left(\sqrt{2}\right)^2(1) = \underline{4}$	Both $y_{\frac{\pi}{4}} = \underline{2}$ and $\left(\frac{dy}{dx}\right)_{\frac{\pi}{4}} = \underline{4}$	B1
	$\left(\frac{d^2y}{dx^2}\right)_{\frac{\pi}{4}} = 6\left(\sqrt{2}\right)^4 - 4\left(\sqrt{2}\right)^2 = 24 - 8 = 16$	Attempts to substitute $x = \frac{\pi}{4}$ into both terms in the expression for $\frac{d^2y}{dx^2}$.	M1
	$\frac{d^3y}{dx^3} = 24\sec^3 x(\sec x \tan x) - 8\sec x(\sec x \tan x)$	Two terms differentiated with either $24\sec^4 x \tan x$ or $-8\sec^2 x \tan x$ being correct	M1
	$= 24\sec^4 x \tan x - 8\sec^2 x \tan x$		
	$\left(\frac{d^2y}{dx^2}\right)_{\frac{\pi}{4}} = 24\left(\sqrt{2}\right)^4(1) - 8\left(\sqrt{2}\right)^2(1) = 96 - 16 = 80$	$\left(\frac{d^3y}{dx^3}\right)_{\frac{\pi}{4}} = \underline{80}$	B1
	$\sec x \approx 2 + 4\left(x - \frac{\pi}{4}\right) + \frac{16}{2}\left(x - \frac{\pi}{4}\right)^2 + \frac{80}{6}\left(x - \frac{\pi}{4}\right)^3 + \ldots$	Applies a Taylor expansion with at least 3 out of 4 terms ft correctly.	M1
		Correct Taylor series expansion.	A1
			[6]
	$\left\{\sec x \approx 2 + 4\left(x - \frac{\pi}{4}\right) + 8\left(x - \frac{\pi}{4}\right)^2 + \frac{40}{3}\left(x - \frac{\pi}{4}\right)^3 + \ldots\right\}$		
			10 marks

Question Number	Scheme		Marks
6.	$w = \dfrac{z}{z+\mathrm{i}}, \quad z = -\mathrm{i}$		
(a)	$w(z+\mathrm{i}) = z \Rightarrow wz + \mathrm{i}w = z \Rightarrow \mathrm{i}w = z - wz$	Complete method of rearranging to make z the subject.	M1
	$\Rightarrow \mathrm{i}w = z(1-w) \Rightarrow z = \dfrac{\mathrm{i}w}{(1-w)}$	$z = \dfrac{\mathrm{i}w}{(1-w)}$	A1 **aef**
	$\lvert z\rvert = 3 \Rightarrow \left\lvert \dfrac{\mathrm{i}w}{1-w}\right\rvert = 3$	Putting $\lvert z$ in terms of their $w\rvert = 3$	dM1
	$\left\{ \lvert \mathrm{i}w\rvert = 3\lvert 1-w\rvert \Rightarrow \lvert w\rvert = 3\lvert w-1\rvert \Rightarrow \lvert w\rvert^2 = 9\lvert w-1\rvert^2 \Rightarrow \lvert u+\mathrm{i}v\rvert^2 = 9\lvert u+\mathrm{i}v-1\rvert^2 \right\}$		
	$\Rightarrow u^2 + v^2 = 9\left[(u-1)^2 + v^2\right]$	Applies $w = u + \mathrm{i}v$, and uses Pythagoras correctly to get an equation in terms of u and v without any i's.	ddM1
		Correct equation.	A1
	$\left\{ \Rightarrow u^2 + v^2 = 9u^2 - 18u + 9 + 9v^2 \Rightarrow 0 = 8u^2 - 18u + 8v^2 + 9 \right\}$		
	$\Rightarrow 0 = u^2 - \frac{9}{4}u + v^2 + \frac{9}{8}$	Simplifies down to $u^2 + v^2 \pm \alpha u \pm \beta v \pm \delta = 0$.	dddM1
	$\Rightarrow \left(u - \frac{9}{8}\right)^2 - \frac{81}{64} + v^2 + \frac{9}{8} = 0$		
	$\Rightarrow \left(u - \frac{9}{8}\right)^2 + v^2 = \frac{9}{64}$		
	{Circle} centre $\left(\frac{9}{8}, 0\right)$, radius $\frac{3}{8}$	One of centre or radius correct. Both centre and radius correct.	A1 A1 **[8]**
(b)		Circle indicated on the Argand diagram in the correct position in follow through quadrants. Ignore plotted coordinates.	B1ft
		Region outside a circle indicated only.	B1
			[2]
			10 marks

Question Number	Scheme		Marks
7.	$y = \lvert x^2 - a^2 \rvert ,\ a > 1$		
(a)		Correct Shape. Ignore cusps. Correct coordinates.	B1 B1
			[2]
(b)	$\lvert x^2 - a^2 \rvert = a^2 - x\ ,\ a > 1$		
	$\{\lvert x \rvert > a\},\quad x^2 - a^2 = a^2 - x$	$x^2 - a^2 = a^2 - x$	M1 aef
	$\Rightarrow x^2 + x - 2a^2 = 0$		
	$\Rightarrow x = \dfrac{-1 \pm \sqrt{1 - 4(1)(-2a^2)}}{2}$	Applies the quadratic formula or completes the square in order to find the roots.	M1
	$\Rightarrow x = \dfrac{-1 \pm \sqrt{1 + 8a^2}}{2}$	Both correct "simplified down" solutions.	A1
	$\{\lvert x \rvert < a\},\quad -x^2 + a^2 = a^2 - x$	$-x^2 + a^2 = a^2 - x$ or $x^2 - a^2 = x - a^2$	M1 aef
	$\{\Rightarrow x^2 - x = 0 \Rightarrow x(x-1) = 0\}$		
	$\Rightarrow x = 0\ ,1$	$x = 0$ $x = 1$	B1 A1
			[6]
(c)	$\lvert x^2 - a^2 \rvert > a^2 - x\ ,\ a > 1$		
	$x < \dfrac{-1 - \sqrt{1 + 8a^2}}{2}\quad \{\text{or}\}\quad x > \dfrac{-1 + \sqrt{1 + 8a^2}}{2}$	x is less than their least value x is greater than their maximum value	B1 ft B1 ft
	$\{\text{or}\}\quad 0 < x < 1$	For $\{\lvert x \rvert < a\}$, Lowest $< x <$ Highest $0 < x < 1$	M1 A1
			[4]
			12 marks

Question Number	Scheme		Marks
8.	$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 2e^{-t}$, $x = 0, \frac{dx}{dt} = 2$ at $t = 0$.		
(a)	AE, $m^2 + 5m + 6 = 0 \Rightarrow (m+3)(m+2) = 0$ $\Rightarrow m = -3, -2$.		
	So, $x_{CF} = Ae^{-3t} + Be^{-2t}$	$Ae^{m_1 t} + Be^{m_2 t}$, where $m_1 \neq m_2$. $Ae^{-3t} + Be^{-2t}$	M1 A1
	$\left\{ x = ke^{-t} \Rightarrow \frac{dx}{dt} = -ke^{-t} \Rightarrow \frac{d^2x}{dt^2} = ke^{-t} \right\}$		
	$\Rightarrow ke^{-t} + 5(-ke^{-t}) + 6ke^{-t} = 2e^{-t} \Rightarrow 2ke^{-t} = 2e^{-t}$ $\Rightarrow k = 1$	Substitutes ke^{-t} into the differential equation given in the question. Finds $k = 1$.	M1 A1
	$\left\{ \text{So, } x_{PI} = e^{-t} \right\}$		
	So, $x = Ae^{-3t} + Be^{-2t} + e^{-t}$	their x_{CF} + their x_{PI}	M1*
	$\frac{dx}{dt} = = -3Ae^{-3t} - 2Be^{-2t} - e^{-t}$	Finds $\frac{dx}{dt}$ by differentiating their x_{CF} and their x_{PI}	dM1*
	$t = 0, x = 0 \Rightarrow \quad 0 = A + B + 1$ $t = 0, \frac{dx}{dt} = 2 \Rightarrow \quad 2 = -3A - 2B - 1$	Applies $t = 0, x = 0$ to x and $t = 0, \frac{dx}{dt} = 2$ to $\frac{dx}{dt}$ to form simultaneous equations.	ddM1*
	$\left\{ \begin{array}{l} 2A + 2B = -2 \\ -3A - 2B = 3 \end{array} \right\}$		
	$\Rightarrow A = -1, B = 0$		
	So, $x = -e^{-3t} + e^{-t}$	$x = -e^{-3t} + e^{-t}$	A1 **cao** **[8]**

Question Number	Scheme		Marks
8.	$x = -\mathrm{e}^{-3t} + \mathrm{e}^{-t}$		
(b)	$\frac{\mathrm{d}x}{\mathrm{d}t} = 3\mathrm{e}^{-3t} - \mathrm{e}^{-t} = 0$	Differentiates their x to give $\frac{\mathrm{d}x}{\mathrm{d}t}$ and puts $\frac{\mathrm{d}x}{\mathrm{d}t}$ equal to 0.	M1
	$3 - \mathrm{e}^{2t} = 0$	A credible attempt to solve.	dM1
	$\Rightarrow t = \frac{1}{2}\ln 3$	$t = \frac{1}{2}\ln 3$ or $t = \ln\sqrt{3}$ or awrt 0.55	A1
	So, $x = -\mathrm{e}^{-\frac{3}{2}\ln 3} + \mathrm{e}^{-\frac{1}{2}\ln 3} = -\mathrm{e}^{\ln 3^{-\frac{3}{2}}} + \mathrm{e}^{\ln 3^{-\frac{1}{2}}}$ $x = -3^{-\frac{3}{2}} + 3^{-\frac{1}{2}}$	Substitutes their t back into x and an attempt to eliminate out the ln's.	ddM1
	$= -\frac{1}{3\sqrt{3}} + \frac{1}{\sqrt{3}} = \frac{2}{3\sqrt{3}} = \underline{\frac{2\sqrt{3}}{9}}$	uses exact values to give $\frac{2\sqrt{3}}{9}$	A1 **AG**
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -9\mathrm{e}^{-3t} + \mathrm{e}^{-t}$ At $t = \frac{1}{2}\ln 3$, $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -9\mathrm{e}^{-\frac{3}{2}\ln 3} + \mathrm{e}^{-\frac{1}{2}\ln 3}$	Finds $\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$ and substitutes their t into $\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$	dddM1
	$= -9(3)^{-\frac{3}{2}} + 3^{-\frac{1}{2}} = -\frac{9}{3\sqrt{3}} + \frac{1}{\sqrt{3}} = -\frac{3}{\sqrt{3}} + \frac{1}{\sqrt{3}}$		
	As $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\frac{9}{3\sqrt{3}} + \frac{1}{\sqrt{3}} = \left\{-\frac{2}{\sqrt{3}}\right\} < 0$ then x is maximum.	$-\frac{9}{3\sqrt{3}} + \frac{1}{\sqrt{3}} < 0$ and maximum conclusion.	A1
			[7]
			15 marks

edexcel

June 2009
6668 Further Pure Mathematics FP2
Appendix

List of Abbreviations

- dM1 denotes a method mark which is dependent upon the award of the previous method mark.
- ddM1 denotes a method mark which is dependent upon the award of the previous two method marks.
- dM1$*$ denotes a method mark which is dependent upon the award of M1$*$.

- ft or $\sqrt{}$ denotes "follow through"
- cao denotes "correct answer only"
- aef denotes "any equivalent form"
- cso denotes "correct solution only"
- AG denotes "answer given" (in the question paper.)
- awrt denotes "anything that rounds to"
- aliter denotes "alternative methods"
- SC denotes "special case"

Extra Solutions

If a candidate makes more than one attempt at any question:

- If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
- If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

Question 1

Question Number	Scheme
1. (a)	
B1	$\frac{1}{2r}-\frac{1}{2(r+2)}$ or $\frac{\frac{1}{2}}{r}-\frac{\frac{1}{2}}{(r+2)}$ aef
(b)	
M1	List the first two terms and the last two $(\ \ )$ terms. Note that A and B must be found. Note that you need to be convinced that they are attempting to substitute $r=1$ and $r=2$ into their series expansion.
M1	Includes (usually adds) the first two underlined terms and includes (usually subtracts) the final two underlined terms from $=\left(\frac{2}{\underline{1}}-\frac{2}{3}\right)+\left(\frac{2}{\underline{2}}-\frac{2}{4}\right)+\ ......+\left(\frac{2}{n-1}-\frac{2}{\underline{n+1}}\right)+\left(\frac{2}{n}-\frac{2}{\underline{n+2}}\right)$ to give for example $\frac{2}{1}+\frac{2}{2};-\frac{2}{n+1}-\frac{2}{n+2}$. Do not award this mark if there are too few terms or any extra non-allowable terms.
A1	For $\frac{2}{1}+\frac{2}{2}-\frac{2}{n+1}-\frac{2}{n+2}$ or $3-\frac{2}{n+1}-\frac{2}{n+2}$.
M1	Attempt to combine to an at least 3 term fraction to a single fraction and an attempt to take out the brackets from their numerator.
A1	Correct result of $\frac{n(3n+5)}{(n+1)(n+2)}$. Note that the answer is given in the question.
	Note that an alternative mark scheme has been produced for those candidates who get their partial fractions the wrong way round in part (a) and then apply them to part (b).

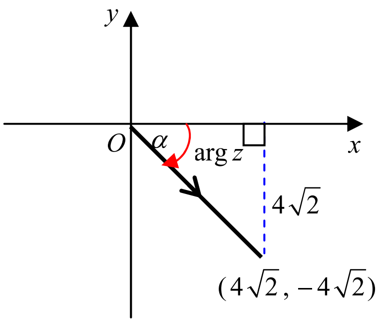
Question Number	Scheme		Marks
Aliter (b) **Way 2**	$\sum_{r=1}^{n} \frac{1}{r(r+2)} = \sum_{r=1}^{n} \left(\frac{1}{2r} - \frac{1}{2(r+2)} \right)$		
	$= \left(\underline{\frac{1}{2}} - \frac{1}{6} \right) + \left(\underline{\frac{1}{4}} - \frac{1}{8} \right) + \ \ldots\ldots$ $\ldots\ldots\ldots\ldots + \left(\frac{1}{2(n-1)} - \underline{\frac{1}{2(n+1)}} \right) + \left(\frac{1}{2n} - \underline{\frac{1}{2(n+2)}} \right)$	List the first two terms and the last two terms	M1
	$= \frac{1}{2} + \frac{1}{4} - \frac{1}{2(n+1)} - \frac{1}{2(n+2)}$	Includes the first two underlined terms and includes the final two underlined terms.	M1
		$\frac{1}{2} + \frac{1}{4} - \frac{1}{2(n+1)} - \frac{1}{2(n+2)}$	A1
	$= \frac{3}{4} - \frac{1}{2(n+1)} - \frac{1}{2(n+2)}$		
	$= \frac{3(n+1)(n+2) - 2(n+2) - 2(n+1)}{4(n+1)(n+2)}$ $= \frac{3n^2 + 9n + 6 - 2n - 4 - 2n - 2}{4(n+1)(n+2)}$	Attempt to combine to an at least 3 term fraction to a single fraction and an attempt to take out the brackets from their numerator.	M1
	$= \frac{3n^2 + 5n}{4(n+1)(n+2)}$		
	$4\sum_{r=1}^{n} \frac{1}{r(r+2)} = 4\left(\frac{3n^2 + 5n}{4(n+1)(n+2)} \right)$		
	$= \frac{n(3n+5)}{(n+1)(n+2)}$	Correct Result	A1 **cso** **AG**
			[5]

Mark Scheme for Alternative "Incorrect" response.

Question Number	Scheme		Marks
1. (a)	$\dfrac{1}{r(r+2)} = \dfrac{1}{2(r+2)} - \dfrac{1}{2r}$	Incorrect answer	B0
			(1)
(b)	$\displaystyle\sum_{r=1}^{n}\frac{4}{r(r+2)} = \sum_{r=1}^{n}\left(\frac{2}{(r+2)} - \frac{2}{r}\right)$		
	$= \left(\frac{2}{3} - \underline{\frac{2}{1}}\right) + \left(\frac{4}{4} - \underline{\frac{2}{2}}\right) + \ \ldots\ldots$ $\ldots\ldots\ldots\ldots + \left(\underline{\frac{2}{n+1}} - \frac{2}{n-1}\right) + \left(\underline{\frac{2}{n+2}} - \frac{2}{n}\right)$	List the first two terms and the last two terms	M1
	$= \dfrac{2}{n+1} + \dfrac{2}{n+2} - \dfrac{2}{1} - \dfrac{2}{2}$	Includes the first two underlined terms and includes the final two underlined terms.	M1
		Cannot award this accuracy mark.	A0
	$= \dfrac{2}{n+1} + \dfrac{2}{n+2} - 3$		
	$= \dfrac{2(n+2) + 2(n+1) - 3(n+1)(n+2)}{(n+1)(n+2)}$ $= \dfrac{2n + 4 + 2n + 2 - 3n^2 - 9n - 6}{(n+1)(n+2)}$	Attempt to combine to an at least 3 term fraction to a single fraction and an attempt to take out the brackets from their numerator.	M1
	$= \dfrac{-3n^2 - 5n}{(n+1)(n+2)}$		
	$= \dfrac{-n(3n+5)}{(n+1)(n+2)}$	Cannot award this accuracy mark.	A0

Question 2

Question Number	Scheme
2. (a)	
M1	A valid attempt to find the modulus and argument of $4\sqrt{2} - 4\sqrt{2}\,\mathrm{i}$. Note there must be an attempt to find both the modulus and argument.
M1	Taking the cube root of the modulus and dividing the argument by 3.
A1	For $2\left(\cos\left(-\frac{\pi}{12}\right) + \mathrm{i}\sin\left(-\frac{\pi}{12}\right)\right)$. Allow $2\mathrm{e}^{-\frac{\pi \mathrm{i}}{12}}$.
M1	**Way 1:** Adding or subtracting 2π to the argument for z^3 in order to find other roots or **Way 2:** Adding or subtracting $\frac{2\pi}{3}$ to the argument for z in order to find the other roots. Note that for Way 2 the candidate needs to divide their argument for z^3 by 3 and then add and subtract multiples of $\frac{2\pi}{3}$ from this new argument.
A1	Any one of $2\left(\cos\frac{7\pi}{12} + \mathrm{i}\sin\frac{7\pi}{12}\right)$ or $2\left(\cos\left(\frac{-3\pi}{4}\right) + \mathrm{i}\sin\left(\frac{-3\pi}{4}\right)\right)$ or $2\mathrm{e}^{\frac{7\pi \mathrm{i}}{12}}$ or $2\mathrm{e}^{-\frac{3\pi \mathrm{i}}{4}}$.
A1	Both $2\left(\cos\frac{7\pi}{12} + \mathrm{i}\sin\frac{7\pi}{12}\right)$ and $2\left(\cos\left(\frac{-3\pi}{4}\right) + \mathrm{i}\sin\left(\frac{-3\pi}{4}\right)\right)$. Do not allow any/both of the roots for *z* expressed in an exponential form for the final mark. Do not award the final A1 mark if there are any extra root(s) within the range $-\pi < \theta \leq \pi$.
	Note that the first accuracy is dependent upon the first two method marks. Note that the final two accuracy marks are dependent on the third method mark. Note that any/both of the first two method marks can be implied.
	There is an alternative method that appears in the appendix. In Way 2, candidates pick up some of the earlier marks later on in their solution. You need to award the marks on ePEN, however, in the same order as they appear for Way 1 on the mark scheme.
	Special Case 1: For a candidate who finds ALL three of $2\left(\cos\frac{\pi}{12} + \mathrm{i}\sin\frac{\pi}{12}\right)$, $2\left(\cos\frac{3\pi}{4} + \mathrm{i}\sin\frac{3\pi}{4}\right)$ and $2\left(\cos\left(\frac{-7\pi}{12}\right) + \mathrm{i}\sin\left(\frac{-7\pi}{12}\right)\right)$ then award SC: M1M1A1M1A0A0.
	Special Case 2: If *r* is incorrect and candidate states the brackets ( ) correctly then give the first accuracy mark ONLY where this is applicable.
	Note that $2\left(\cos\frac{\pi}{12} - \mathrm{i}\sin\frac{\pi}{12}\right)$ or $2\left(\cos\left(-\frac{\pi}{12}\right) - \mathrm{i}\sin\frac{\pi}{12}\right)$ are also fine for the first accuracy mark. Note also that $2\left(\cos\frac{3\pi}{4} - \mathrm{i}\sin\frac{3\pi}{4}\right)$ or $2\left(\cos\left(\frac{-3\pi}{4}\right) - \mathrm{i}\sin\frac{3\pi}{4}\right)$ are also fine for the second accuracy mark. Withhold the final accuracy mark, however, if all three roots are not in the form $r\left(\cos\theta + \mathrm{i}\sin\theta\right)$. Note that there needs to be a "+" between $\cos\theta$ and $\mathrm{i}\sin\theta$ and the angle θ must be consistent (ie. the same) for cos and sin inside the brackets for the award of the final accuracy mark.

Question Number	Scheme		Marks
2. (a)	$z^3 = 4\sqrt{2} - 4\sqrt{2}\,\mathrm{i}\ ,\ \ -\pi < \theta \leqslant \pi$		
			
	$r = \sqrt{\left(4\sqrt{2}\right)^2 + \left(-4\sqrt{2}\right)^2} = \sqrt{32+32} = \sqrt{64} = 8$ $\theta = -\tan^{-1}\left(\frac{4\sqrt{2}}{4\sqrt{2}}\right) = -\frac{\pi}{4}$	A valid attempt to find the modulus and argument of $4\sqrt{2} - 4\sqrt{2}\,\mathrm{i}$.	M1
	$z^3 = 8\left(\cos\left(-\frac{\pi}{4}\right) + \mathrm{i}\sin\left(-\frac{\pi}{4}\right)\right)$		
		Decide to award M1 here!! ***Decide to award A1 here!!***	M1 A1
	$z^3 = 8\left(\cos\left(-\frac{\pi}{4} + 2k\pi\right) + \mathrm{i}\sin\left(-\frac{\pi}{4} + 2k\pi\right)\right)$	Adding or subtracting $2(k)\pi$ to the argument for z^3 in order to find other roots.	M1
	So, $z = (8)^{\frac{1}{3}}\left(\cos\left(\frac{-\frac{\pi}{4} + 2k\pi}{3}\right) + \mathrm{i}\sin\left(\frac{-\frac{\pi}{4} + 2k\pi}{3}\right)\right)$	Taking the cube root of the modulus and dividing the argument by 3.	**Award above**
	$\Rightarrow z = 2\left(\cos\left(-\frac{\pi}{12} + \frac{2k\pi}{3}\right) + \mathrm{i}\sin\left(-\frac{\pi}{12} + \frac{2k\pi}{3}\right)\right)$		
	$k = 0,\ \ z = 2\left(\cos\left(-\frac{\pi}{12}\right) + \mathrm{i}\sin\left(-\frac{\pi}{12}\right)\right)$	$2\left(\cos\left(-\frac{\pi}{12}\right) + \mathrm{i}\sin\left(-\frac{\pi}{12}\right)\right)$	**Award Above**
	$k = 1,\ \ z = 2\left(\cos\frac{7\pi}{12} + \mathrm{i}\sin\frac{7\pi}{12}\right)$	Any one of the final two roots	A1
	$k = -1,\ \ z = 2\left(\cos\left(\frac{-3\pi}{4}\right) + \mathrm{i}\sin\left(\frac{-3\pi}{4}\right)\right)$	Both of the final two roots.	A1
			[6]
			6 marks

Special Case 1: Award SC: M1M1A1M1A0A0 for ALL three of $2\left(\cos\frac{\pi}{12} + \mathrm{i}\sin\frac{\pi}{12}\right)$, $2\left(\cos\frac{3\pi}{4} + \mathrm{i}\sin\frac{3\pi}{4}\right)$ and $2\left(\cos\left(\frac{-7\pi}{12}\right) + \mathrm{i}\sin\left(\frac{-7\pi}{12}\right)\right)$.

Special Case 2: If r is incorrect and candidate states the brackets ( ) correctly then give the first accuracy mark ONLY where this is applicable.

Question 3

Question Number	Scheme
3. (a)	
M1	An attempt to divide every term in the differential equation by $\sin x$. Can be implied. You can be generous with the result when the candidate divides $-y\cos x$ by $\sin x$.
	Hence, the resulting DE must be in the form $\frac{\mathrm{d}y}{\mathrm{d}x} \pm \mathrm{P}(x)y = \frac{\sin 2x \sin x}{\sin x}$, where $\mathrm{P}(x)$ is a function of x.
	For example, $\mathrm{P}(x)$ can be $\tan x$.
	You can imply the first M1 mark if the candidate writes down an integrating factor of the form $e^{\int \pm \frac{\cos x}{\sin x}(\mathrm{d}x)}$.
M1	For applying an integrating factor $e^{\int \pm \frac{\cos x}{\sin x}(\mathrm{d}x)}$ or $e^{\int \text{their } \mathrm{P}(x)\,(\mathrm{d}x)}$. Ignore $\mathrm{d}x$.
A1	$e^{-\ln \sin x}$ or $e^{\ln \operatorname{cosec} x}$ or any equivalent.
A1	$\frac{1}{\sin x}$ or $(\sin x)^{-1}$ or $\operatorname{cosec} x$
M1	$\frac{\mathrm{d}}{\mathrm{d}x}(y \times \text{their I.F.}) = \sin 2x \times \text{their I.F}$ or $(y \times \text{their I.F.}) = \int \sin 2x \times \text{their I.F}\,(dx)$.
A1	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{y}{\sin x}\right) = 2\cos x$ or $\frac{y}{\sin x} = \int 2\cos x\,(\mathrm{d}x)$, ignoring the omission of $\mathrm{d}x$.
dddM1	A credible attempt to integrate the RHS with/without $+K$. Note that this mark is dependent upon the candidate receiving the first three method marks.
A1	$y = 2\sin^2 x + K\sin x$, **cao.**
	If the candidate writes down the integrating factor of either $e^{-\ln \sin x}$ or $e^{\ln \operatorname{cosec} x}$ (with no working), then you can imply the first two method marks.

Question 4

Question Number	Scheme
4. (a)	
B1	For $\frac{1}{2}\int_0^{2\pi}(a+3\cos\theta)^2\,\mathrm{d}\theta$, with correct limits and ignoring $\mathrm{d}\theta$.
	Note that $a^2+9\cos^2\theta$ is acceptable for r^2.
M1	Replacing $\cos^2\theta$ with an expression of the form $\cos^2\theta=\frac{\pm1\pm\cos2\theta}{2}$.
A1	Achieving $\underline{a^2+6a\cos\theta+9\left(\frac{1+\cos2\theta}{2}\right)}$.
	The underlined expression can appear over a few lines of a candidate's working.
M1*	Integrate to get an expression with at least 3 out of 4 terms in the form $\pm A\theta\pm B\sin\theta\pm C\theta\pm D\sin2\theta$.
	Ignore the $\frac{1}{2}$ and ignore the limits.
A1ft	$a^2\theta+6a\sin\theta+$ correct ft integration. Ignore the $\frac{1}{2}$ and ignore the limits.
A1	For $\pi a^2+\frac{9\pi}{2}$.
dM1*	Candidates need to put their integrated expression equal to $\frac{107}{2}\pi$.
	Note that is method mark is dependent upon the M1* mark being awarded.
A1	$a=7$. Do not allow $a=\pm7$ without reference to a being equal to 7.

Question 5

Note that the marks in question 5 are now: (a) B1 M1 A1 A1 (b) B1 M1 M1 **B1** M1 A1

Question Number	Scheme
5. (a)	
B1	Correct differentiation of $y = \sec^2 x$.
	Examples that are acceptable are $2(\sec x)^1(\sec x \tan x)$ or $2\sec^2 x \tan x$ or $-2(\cos x)^{-2}(-\sin x)$ or $2\sin x(\cos x)^{-2}$ or $\dfrac{2\sin x}{\cos^2 x}$, etc.
M1	Two terms added with one of either $A\sec^2 x \tan^2 x$ or $B\sec^4 x$ in the correct form.
	Special Case: M1 can also be awarded in this part if candidate differentiates $y' = 2\sec x \tan x$ to give the "correct" answer of $y'' = 2\sec^3 x + 2\sec x \tan^2 x$.
A1	Correct differentiation of give $4\sec^2 x \tan^2 x + 2\sec^4 x$.
A1	Applies $\tan^2 x = \sec^2 x - 1$ leading to the correct result. Note that the answer is given in the question.
(b)	
B1	Both $y_{\frac{\pi}{4}} = \underline{2}$ and $\left(\dfrac{dy}{dx}\right)_{\frac{\pi}{4}} = \underline{4}$.
M1	Attempts to substitute $x = \frac{\pi}{4}$ (or even $x = -\frac{\pi}{4}$) into both terms in the expression for $\dfrac{d^2y}{dx^2}$ given in the question or even their expression for $\dfrac{d^2y}{dx^2}$.
	$6\left(\sqrt{2}\right)^4 - 4\left(\sqrt{2}\right)^2$ or $6\sec^4\left(\frac{\pi}{4}\right) - 4\sec^2\left(\frac{\pi}{4}\right)$ are sufficient for the award of M1 here.
	Note that $f''(\frac{\pi}{4})$ by itself is not sufficient.
M1	Two terms with either $24\sec^4 x \tan x$ or $-8\sec^2 x \tan x$ being correct.
B1	$\left(\dfrac{d^3y}{dx^3}\right)_{\frac{\pi}{4}} = \underline{80}$
M1	Applies a Taylor expansion $f\left(\frac{\pi}{4}\right) + \left(x - \frac{\pi}{4}\right)f'\left(\frac{\pi}{4}\right) + \left(x - \frac{\pi}{4}\right)^2 \dfrac{f''\left(\frac{\pi}{4}\right)}{2!} + \left(x - \frac{\pi}{4}\right)^3 \dfrac{f'''\left(\frac{\pi}{4}\right)}{3!} + \ldots$ with at least 3 out of 4 terms ***followed through correctly*** with their $f\left(\frac{\pi}{4}\right), f'\left(\frac{\pi}{4}\right), f''\left(\frac{\pi}{4}\right), f'''\left(\frac{\pi}{4}\right)$.
A1	Correct Taylor series expansion of $2 + 4\left(x - \frac{\pi}{4}\right) + \frac{16}{2}\left(x - \frac{\pi}{4}\right)^2 + \frac{80}{6}\left(x - \frac{\pi}{4}\right)^3 + \ldots$

Question 6

Question Number	Scheme
6. (a)	
M1	Complete method of rearranging to make z the subject.
A1	$z = \frac{\mathrm{i}w}{(1-w)}$ or $z = \frac{-\mathrm{i}w}{(w-1)}$
dM1	Putting $\lvert z \text{ in terms of their } w \rvert = 3$. Note that this mark is dependent upon the first M1 mark being awarded.
ddM1	Applies $w = u + \mathrm{i}v$, and uses Pythagoras correctly (on grouped real and imaginary parts) to get an equation in terms of u and v without any i's. Effectively they need "to square and add". Note that $u^2 + v^2 = 3\left[(u-1)^2 + v^2\right]$ would be fine for M1. (Allow working in x and y where candidate applies $w = x + \mathrm{i}y$.) Note that this mark is dependent upon the first two M1 marks being awarded.
A1	Correct equation. Aef, for example $u^2 + v^2 = 9\left[(1-u)^2 + v^2\right]$.
dddM1	Simplifies down to $u^2 + v^2 \pm \alpha u \pm \beta v \pm \delta = 0$. Note this needs to be an EQUATION, with the coefficients of u^2 and v^2 either both 1 or both -1, but u^2 and v^2 need to be on the same side. Also note that equations of the form $u^2 + v^2 = \pm\alpha u \pm \beta v \pm \delta$, $u^2 + v^2 \pm \alpha u = \pm \beta v \pm \delta$ are acceptable for M1. Note that this mark is dependent upon the first three M1 marks being awarded.
A1	One of centre $\left(\frac{9}{8}, 0\right)$ or radius $\frac{3}{8}$ correct.
A1	Both the centre $\left(\frac{9}{8}, 0\right)$ and radius $\frac{3}{8}$ correct.
(b)	
B1ft	Circle indicated on the Argand diagram in the correct position in follow through quadrants. This means that candidates must have stated either (however, incorrect) the equation of the circle or the centre and radius of the circle in either parts (a) or (b). Also DO NOT allow the B1 mark if a candidate draws a circle centre $(0, 0)$. Ignore plotted coordinates.
B1	Region outside a circle indicated only. If a candidate draws a circle and shades in the region outside of it then this is OK for B1. The candidate for this mark DOES NOT have to state their equation of the circle or their centre and radius of the circle in either parts (a) or (b).

<table>
<tr><th>Question Number</th><th>Example</th></tr>
<tr><td>6. (b)
EG 1</td><td>

$u^2 - \frac{1}{4}u + v^2 = \frac{1}{8}$

$(u - \frac{1}{8})^2 - \frac{1}{64} + v^2 = \frac{1}{8} \quad \therefore (u - \frac{1}{8})^2 + v^2 = \frac{9}{64}$

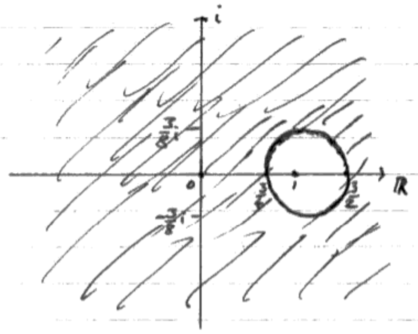
$\therefore \frac{9}{64} = r^2$

radius $= \frac{3}{8}$

b) $|z| < 3 \qquad |z| = \frac{|iw|}{|w-1|}$

$\therefore \frac{|iw|}{|w-1|} < 3$

$|w| < 3|w-1|$



shaded region is R

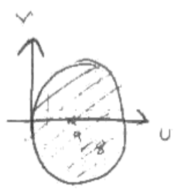
Comment: In (b), this response would score B0 B1.
</td></tr>
<tr><td>EG 2</td><td>

Question 6 continued

$\Rightarrow (u - \frac{9}{8})^2 + v^2 = \frac{9}{64} \quad \therefore$ circle centre $(\frac{9}{8}, 0)$ radius $= \frac{3}{8}$.

$|w \cdot|$

b.)



Comment: In (a), this response would score Way 2: B0 B1, as the circle should not be touching the v-axis.
</td></tr>
</table>

Question 7

Note that the marks in question 7 are now: (a) B1 B1 (b) M1 M1 A1 M1 **B1** A1 (c) B1 B1 M1 A1

Question Number	Scheme
7. (a)	
B1	Correct Shape. Ignore cusps.
B1	Note you can award this mark if they state all three of $-a$, a and a^2 on the curve in the correct place. Note that the coordinates must be stated as $(-a, 0)$, $(a, 0)$ and $(0, a^2)$ if they are not referred (marked on) the graph. Accept, however, the coordinates interchanged for x and y if they are marked on the curve in the correct place.
(b)	
M1	Taking out the modulus and writing down $x^2 - a^2 = a^2 - x$ is sufficient for this mark.
M1	Applies the quadratic formula with correct "a, b and c" for their equation. Or completes the square in order to find the roots. If they decide to complete the square it must be done on $x^2 + x$. So $\left(x + \frac{1}{2}\right)^2 - \frac{1}{4} + 2a^2 = 0$ and an attempt to solve for x is sufficient for M1.
A1	Both correct "simplified down" solutions of $\frac{-1 \pm \sqrt{1 + 8a^2}}{2}$ or $-\frac{1}{2} \pm \frac{\sqrt{1 + 8a^2}}{2}$ or $-\frac{1}{2} \pm \sqrt{\frac{1}{4} + 2a^2}$, etc.
M1	Either $-x^2 + a^2 = a^2 - x$ or $x^2 - a^2 = x - a^2$.
B1	Answer of $x = 0$ seen anywhere in part (b). Note that this mark is independent of method.
A1	$x = 1$. Note that this mark is only dependent on the third method mark being awarded.
	If the candidate gives all four solutions of $x = \frac{-1 \pm \sqrt{1 + 8a^2}}{2}$, 0, 1 and includes any extra additional solutions then deduct the final accuracy mark. Note that the candidate cannot get full marks in this part for additional extra solutions. If a candidate rejects a solution then do not mark the rejected solution. Note that an alternative method of squaring both sides is detailed in this appendix.
(c)	
B1ft	x is less than their least value. Note that this is a follow through mark.
B1ft	x is greater than their maximum value. Note that this is a follow through mark.
M1	When the "negative" modulus is taken $\{\lvert x \rvert < a\}$, Lowest Solution < x < Highest Solution
A1	$0 < x < 1$
	Note that the greatest value of x is the largest one of the two solutions found when the positive modulus (for $\lvert x \rvert > a$) is taken and the least value of x is smallest one of the two solutions found when the positive modulus (for $\lvert x \rvert > a$) is taken. **SC 1**: For x is less than or equal their least value and x is greater than or equal their maximum value then award B1B0. **SC 2**: For $0 \le x \le 1$, then award M1 A0.

Question Number	Scheme		Marks
Aliter			
(b)	$\lvert x^2 - a^2 \rvert = a^2 - x\ ,\ a > 1$		
Way 2			
	$(x^2 - a^2)^2 = (a^2 - x)^2$	$(x^2 - a^2)^2 = (a^2 - x)^2$	M1
	$x^4 - 2a^2x^2 + a^4 = a^4 - 2a^2x + x^2$		
	$x^4 - 2a^2x^2 + a^4 = a^4 - 2a^2x + x^2$		
	$x^4 - 2a^2x^2 + a^4 - a^4 + 2a^2x - x^2 = 0$	Rearranges equation onto one side = 0	M1
	$x^4 - 2a^2x^2 - x^2 + 2a^2x = 0$		
	$x(x^3 - 2a^2x - x + 2a^2) = 0$		
	Hence, $x = 0$	$x = 0$ appearing anywhere in part (b)	B1
	So, $(x^3 - 2a^2x - x + 2a^2) = 0$		
	$(x^3 - (2a^2 + 1)x + 2a^2) = 0$		
	$(x - 1)(x^2 + x - 2a^2) = 0$	Factorises $(x - 1)$ from their cubic expression.	M1
	Hence, $x = 1$	$x = 1$	A1
	$x = \dfrac{-1 \pm \sqrt{1 - 4(1)(-2a^2)}}{2}$		
	$x = \dfrac{-1 \pm \sqrt{1 + 8a^2}}{2}$	Both correct "simplified down" solutions.	A1
			[6]

For Way 2, the marks are M1 M1 B1 M1 A1 A1. They should be awarded **in this order** on ePEN.

Question 8

Question Number	Scheme
8. (a)	
M1	Writes down a solution in the form, $Ae^{m_1 t} + Be^{m_2 t}$, where $m_1 \neq m_2$. Note that they must put a constant in front of each exponential term.
A1	$Ae^{-3t} + Be^{-2t}$. Aef, which usually means $Ae^{-2t} + Be^{-3t}$.
M1	Substitutes ke^{-t} into the differential equation given in the question.
A1	Finds $k = 1$.
M1*	their x_{CF} + their x_{PI}. They must add together their x_{CF} and their x_{PI} which they have found. Allow this mark generously they have found $k = 1$ and then write $Ae^{-2t} + Be^{-3t} + 1$.
dM1*	Finds $\frac{dx}{dt}$ by differentiating their x_{CF} and their x_{PI}. Note that this method mark is dependent upon the award of M1*.
ddM1*	Applies $t = 0,\ x = 0$ to x and $t = 0, \frac{dx}{dt} = 2$ to $\frac{dx}{dt}$ to form simultaneous equations. Note that this method mark is dependent upon the award of the previous 2 method marks.
A1	For the correct answer of $x = -e^{-3t} + e^{-t}$.
(b)	
M1	Differentiates their x to give $\frac{dx}{dt}$ and puts $\frac{dx}{dt}$ equal to 0.
dM1	A credible attempt to solve. Multiplying through by e^{3t} to give $3 - e^{2t} = 0$ or trying to form a quadratic in e^{-t} and factorising this quadratic or using the formula on this quadratic is enough for the method mark here.
A1	$t = \frac{1}{2}\ln 3$ or $t = \ln\sqrt{3}$ or awrt 0.55
ddM1	Substitutes their t back into x and an attempt to eliminate out the ln's.
A1	Uses exact values to give $\frac{2\sqrt{3}}{9}$.
dddM1	Finds $\frac{d^2x}{dt^2}$ and substitutes their t into $\frac{d^2x}{dt^2}$
A1	$-\frac{9}{3\sqrt{3}} + \frac{1}{\sqrt{3}}$ or $-\frac{3}{\sqrt{3}} + \frac{1}{\sqrt{3}}$ or $-\frac{2}{\sqrt{3}}$ or $-\frac{2\sqrt{3}}{3}$ or awrt -1.2 and < 0 and maximum conclusion.
	In part (b), each method mark is dependent upon all previous method marks being awarded.

Alternative Way with constants A and B the other way round.

Question Number	Scheme		Marks
8.	$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 2e^{-t}$, $x = 0, \frac{dx}{dt} = 2$ at $t = 0$.		
(a)	AE, $m^2 + 5m + 6 = 0 \Rightarrow (m+3)(m+2) = 0$ $\Rightarrow m = -2, -3$.		
	So, $x_{CF} = Ae^{-2t} + Be^{-3t}$	$Ae^{m_1 t} + Be^{m_2 t}$, where $m_1 \neq m_2$. $Ae^{-2t} + Be^{-3t}$	M1 A1
	$\left\{ x = ke^{-t} \Rightarrow \frac{dx}{dt} = -ke^{-t} \Rightarrow \frac{d^2x}{dt^2} = ke^{-t} \right\}$		
	$\Rightarrow ke^{-t} + 5(-ke^{-t}) + 6ke^{-t} = 2e^{-t} \Rightarrow 2ke^{-t} = 2e^{-t}$ $\Rightarrow k = 1$	Substitutes ke^{-t} into the differential equation given in the question. Finds $k = 1$.	M1 A1
	$\left\{ \text{So, } x_{PI} = e^{-t} \right\}$		
	So, $x = Ae^{-2t} + Be^{-3t} + e^{-t}$	their x_{CF} + their x_{PI}	M1*
	$\frac{dx}{dt} = = -2Ae^{-2t} - 3Be^{-3t} - e^{-t}$	Finds $\frac{dx}{dt}$ by differentiating their x_{CF} and their x_{PI}	dM1*
	$t = 0,\ x = 0 \Rightarrow \quad 0 = A + B + 1$ $t = 0, \frac{dx}{dt} = 2 \Rightarrow \quad 2 = -2A - 3B - 1$	Applies $t = 0,\ x = 0$ to x and $t = 0, \frac{dx}{dt} = 2$ to $\frac{dx}{dt}$ to form simultaneous equations.	ddM1*
	$\left\{ \begin{array}{l} 2A + 2B = -2 \\ -2A - 3B = 3 \end{array} \right\}$		
	$\Rightarrow A = 0, B = -1$		
	So, $x = -e^{-3t} + e^{-t}$	$x = -e^{-3t} + e^{-t}$	A1 **[8]**