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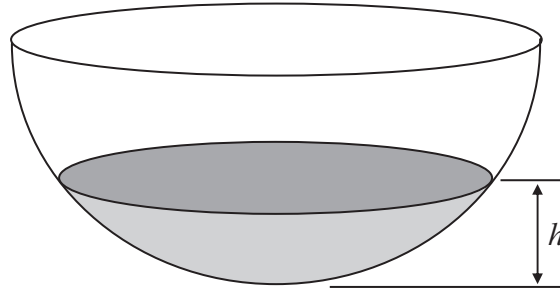


Figure 1

A hollow hemispherical bowl is shown in Figure 1. Water is flowing into the bowl. When the depth of the water is  $h$  m, the volume  $V$  m<sup>3</sup> is given by

$$V = \frac{1}{12} \pi h^2 (3 - 4h), \quad 0 \leq h \leq 0.25$$

(a) Find, in terms of  $\pi$ ,  $\frac{dV}{dh}$  when  $h = 0.1$  (4)

Water flows into the bowl at a rate of  $\frac{\pi}{800}$  m<sup>3</sup>s<sup>-1</sup>.

(b) Find the rate of change of  $h$ , in m s<sup>-1</sup>, when  $h = 0.1$  (2)

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**Question 3 continued**

Lined writing area for the answer to Question 3.

**(Total 6 marks)**

**Q3**



4.

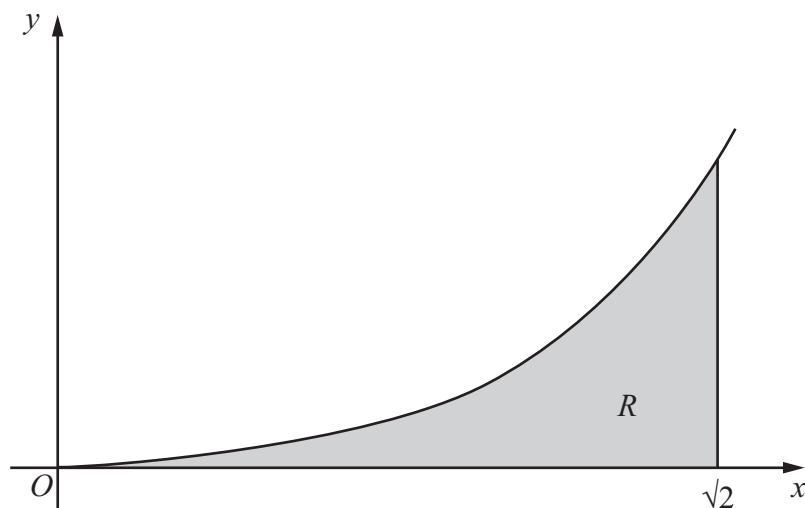


Figure 2

Figure 2 shows a sketch of the curve with equation  $y = x^3 \ln(x^2 + 2)$ ,  $x \geq 0$ . The finite region  $R$ , shown shaded in Figure 2, is bounded by the curve, the  $x$ -axis and the line  $x = \sqrt{2}$ .

The table below shows corresponding values of  $x$  and  $y$  for  $y = x^3 \ln(x^2 + 2)$ .

$x$	0	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{3\sqrt{2}}{4}$	$\sqrt{2}$
$y$	0		0.3240		3.9210

(a) Complete the table above giving the missing values of  $y$  to 4 decimal places. (2)

(b) Use the trapezium rule, with all the values of  $y$  in the completed table, to obtain an estimate for the area of  $R$ , giving your answer to 2 decimal places. (3)

(c) Use the substitution  $u = x^2 + 2$  to show that the area of  $R$  is

$$\frac{1}{2} \int_2^4 (u - 2) \ln u \, du \quad (4)$$

(d) Hence, or otherwise, find the exact area of  $R$ . (6)











5. Find the gradient of the curve with equation

$$\ln y = 2x \ln x, \quad x > 0, y > 0$$

at the point on the curve where  $x = 2$ . Give your answer as an exact value.

(7)

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6. With respect to a fixed origin  $O$ , the lines  $l_1$  and  $l_2$  are given by the equations

$$l_1: \mathbf{r} = \begin{pmatrix} 6 \\ -3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}, \quad l_2: \mathbf{r} = \begin{pmatrix} -5 \\ 15 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix},$$

where  $\lambda$  and  $\mu$  are scalar parameters.

(a) Show that  $l_1$  and  $l_2$  meet and find the position vector of their point of intersection  $A$ . (6)

(b) Find, to the nearest  $0.1^\circ$ , the acute angle between  $l_1$  and  $l_2$ . (3)

The point  $B$  has position vector  $\begin{pmatrix} 5 \\ -1 \\ 1 \end{pmatrix}$ .

(c) Show that  $B$  lies on  $l_1$ . (1)

(d) Find the shortest distance from  $B$  to the line  $l_2$ , giving your answer to 3 significant figures. (4)

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7.

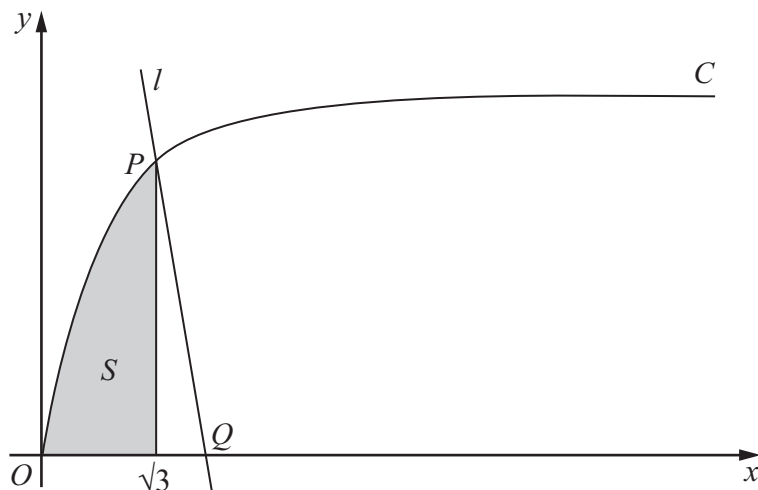


Figure 3

Figure 3 shows part of the curve  $C$  with parametric equations

$$x = \tan \theta, \quad y = \sin \theta, \quad 0 \leq \theta < \frac{\pi}{2}$$

The point  $P$  lies on  $C$  and has coordinates  $\left(\sqrt{3}, \frac{1}{2}\sqrt{3}\right)$ .

(a) Find the value of  $\theta$  at the point  $P$ . (2)

The line  $l$  is a normal to  $C$  at  $P$ . The normal cuts the  $x$ -axis at the point  $Q$ .

(b) Show that  $Q$  has coordinates  $(k\sqrt{3}, 0)$ , giving the value of the constant  $k$ . (6)

The finite shaded region  $S$  shown in Figure 3 is bounded by the curve  $C$ , the line  $x = \sqrt{3}$  and the  $x$ -axis. This shaded region is rotated through  $2\pi$  radians about the  $x$ -axis to form a solid of revolution.

(c) Find the volume of the solid of revolution, giving your answer in the form  $p\pi\sqrt{3} + q\pi^2$ , where  $p$  and  $q$  are constants. (7)

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