

GCE

Edexcel GCE

Core Mathematics C4 (6666)

June 2006

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Mark Scheme  
(Final)

June 2006  
6666 Pure Mathematics C4  
Mark Scheme

Question Number	Scheme	Marks	
1.	$\left\{ \begin{array}{l} \frac{dy}{dx} \\ \frac{dy}{dx} \end{array} \right\} \times \quad 6x - 4y \frac{dy}{dx} + 2 - 3 \frac{dy}{dx} = 0$ $\left\{ \frac{dy}{dx} = \frac{6x+2}{4y+3} \right\}$ <p>At (0, 1), <math>\frac{dy}{dx} = \frac{0+2}{4+3} = \frac{2}{7}</math></p> <p>Hence <math>m(\mathbf{N}) = -\frac{7}{2}</math> or <math>-\frac{1}{\frac{2}{7}}</math></p> <p>Either <math>\mathbf{N}: y - 1 = -\frac{7}{2}(x - 0)</math> or <math>\mathbf{N}: y = -\frac{7}{2}x + 1</math></p> <p><math>\mathbf{N}: 7x + 2y - 2 = 0</math></p>	<p>Differentiates implicitly to include either <math>\pm ky \frac{dy}{dx}</math> or <math>\pm 3 \frac{dy}{dx}</math>. (Ignore <math>\left( \frac{dy}{dx} = \right)</math>.) Correct equation.</p> <p><i>not necessarily required.</i></p> <p>Substituting <math>x = 0</math> &amp; <math>y = 1</math> into an equation involving <math>\frac{dy}{dx}</math>; to give <math>\frac{2}{7}</math> or <math>-\frac{2}{7}</math></p> <p>Uses <math>m(\mathbf{T})</math> to 'correctly' find <math>m(\mathbf{N})</math>. Can be ft from "their tangent gradient".</p> <p><math>y - 1 = m(x - 0)</math> with 'their tangent or normal gradient'; or uses <math>y = mx + 1</math> with 'their tangent or normal gradient';</p> <p>Correct equation in the form '<math>ax + by + c = 0</math>', where a, b and c are integers.</p>	<p>M1 A1</p> <p>dM1; A1 <b>cso</b></p> <p>A1<math>\sqrt{\phantom{x}}</math> oe.</p> <p>M1;</p> <p>A1 oe <b>cso</b></p> <p>[7]</p>
		<b>7 marks</b>	

**Beware:**  $\frac{dy}{dx} = \frac{2}{7}$  does not necessarily imply the award of all the first four marks in this question.

So please ensure that you check candidates' initial differentiation before awarding the first A1 mark.

**Beware:** The final accuracy mark is for completely correct solutions. If a candidate flukes the final line then they must be awarded A0.

**Beware:** A candidate finding an  $m(\mathbf{T}) = 0$  can obtain A1ft for  $m(\mathbf{N}) = \infty$ , but obtains M0 if they write  $y - 1 = \infty(x - 0)$ . If they write, however,  $\mathbf{N}: x = 0$ , then can score M1.

**Beware:** A candidate finding an  $m(\mathbf{T}) = \infty$  can obtain A1ft for  $m(\mathbf{N}) = 0$ , and also obtains M1 if they write  $y - 1 = 0(x - 0)$  or  $y = 1$ .

**Beware:** The final **cso** refers to the whole question.

Question Number	Scheme	Marks
<p><b>Aliter</b></p> <p>1.</p> <p><b>Way 2</b></p>	$\left\{ \begin{array}{l} \cancel{6x} \\ \cancel{4y} \end{array} \right\} \times \left\{ \begin{array}{l} \cancel{2} \\ \cancel{3} \end{array} \right\} \times 6x \frac{dx}{dy} - 4y + 2 \frac{dx}{dy} - 3 = 0$ $\left\{ \frac{dx}{dy} = \frac{4y+3}{6x+2} \right\}$ <p>At (0, 1), <math>\frac{dx}{dy} = \frac{4+3}{0+2} = \frac{7}{2}</math></p> <p>Hence <math>m(\mathbf{N}) = -\frac{7}{2}</math> or <math>-\frac{1}{\frac{2}{7}}</math></p> <p>Either <math>\mathbf{N}: y - 1 = -\frac{7}{2}(x - 0)</math> or <math>\mathbf{N}: y = -\frac{7}{2}x + 1</math></p> <p><math>\mathbf{N}: 7x + 2y - 2 = 0</math></p>	<p>Differentiates implicitly to include either <math>\pm kx \frac{dx}{dy}</math> or <math>\pm 2 \frac{dx}{dy}</math>. (Ignore <math>\left( \frac{dx}{dy} = \right)</math>.)</p> <p>Correct equation.</p> <p><i>not necessarily required.</i></p> <p>Substituting <math>x = 0</math> &amp; <math>y = 1</math> into an equation involving <math>\frac{dx}{dy}</math>; to give <math>\frac{7}{2}</math></p> <p>Uses <math>m(\mathbf{T})</math> or <math>\frac{dx}{dy}</math> to 'correctly' find <math>m(\mathbf{N})</math>. Can be ft using "<math>-1 \cdot \frac{dx}{dy}</math>".</p> <p><math>y - 1 = m(x - 0)</math> with 'their tangent, <math>\frac{dx}{dy}</math> or normal gradient'; or uses <math>y = mx + 1</math> with 'their tangent, <math>\frac{dx}{dy}</math> or normal gradient' ;</p> <p>Correct equation in the form '<math>ax + by + c = 0</math>', where a, b and c are integers.</p> <p>M1 A1</p> <p>dM1; A1 <b>cs</b></p> <p>A1√ oe.</p> <p>M1;</p> <p>A1 oe <b>cs</b></p> <p><b>7 marks</b></p>

Question Number	Scheme	Marks
<p><b>Aliter</b></p> <p><b>1.</b></p> <p><b>Way 3</b></p>	$2y^2 + 3y - 3x^2 - 2x - 5 = 0$ $\left(y + \frac{3}{4}\right)^2 - \frac{9}{16} = \frac{3x^2}{2} + x + \frac{5}{2}$ $y = \sqrt{\left(\frac{3x^2}{2} + x + \frac{49}{16}\right)} - \frac{3}{4}$ $\frac{dy}{dx} = \frac{1}{2} \left(\frac{3x^2}{2} + x + \frac{49}{16}\right)^{-\frac{1}{2}} (3x + 1)$ <p>At (0, 1),</p> $\frac{dy}{dx} = \frac{1}{2} \left(\frac{49}{16}\right)^{-\frac{1}{2}} = \frac{1}{2} \left(\frac{4}{7}\right) = \frac{2}{7}$ <p>Hence <math>m(\mathbf{N}) = -\frac{7}{2}</math></p> <p>Either <math>\mathbf{N}: y - 1 = -\frac{7}{2}(x - 0)</math></p> <p>or <math>\mathbf{N}: y = -\frac{2}{7}x + 1</math></p> <p><math>\mathbf{N}: 7x + 2y - 2 = 0</math></p>	<p>M1;</p> <p>A1 oe</p> <p>dM1</p> <p>A1 <b>cs</b></p> <p>A1√</p> <p>M1</p> <p>A1 oe</p> <p>[7]</p> <p><b>7 marks</b></p>

Question Number	Scheme	Marks
2. (a)	$3x - 1 \equiv A(1 - 2x) + B$ <p>Let <math>x = \frac{1}{2}</math>; <math>\frac{3}{2} - 1 = B \Rightarrow B = \frac{1}{2}</math></p> <p>Equate x terms; <math>3 = -2A \Rightarrow A = -\frac{3}{2}</math></p> <p><b>(No working seen, but A and B correctly stated <math>\Rightarrow</math> award all three marks. If one of A or B correctly stated give two out of the three marks available for this part.)</b></p>	<p>Considers this identity and either substitutes <math>x = \frac{1}{2}</math>, equates coefficients or solves simultaneous equations</p> <p><i>complete</i></p> <p>M1</p> <p>A1;A1</p> <p><b>[3]</b></p>
(b)	$f(x) = -\frac{3}{2}(1 - 2x)^{-1} + \frac{1}{2}(1 - 2x)^{-2}$ $= -\frac{3}{2} \left\{ 1 + (-1)(-2x) + \frac{(-1)(-2)}{2!}(-2x)^2 + \frac{(-1)(-2)(-3)}{3!}(-2x)^3 + \dots \right\}$ $+ \frac{1}{2} \left\{ 1 + (-2)(-2x) + \frac{(-2)(-3)}{2!}(-2x)^2 + \frac{(-2)(-3)(-4)}{3!}(-2x)^3 + \dots \right\}$ $= -\frac{3}{2} \{1 + 2x + 4x^2 + 8x^3 + \dots\} + \frac{1}{2} \{1 + 4x + 12x^2 + 32x^3 + \dots\}$ $= -1 - x + 0x^2 + 4x^3$	<p>Moving powers to top on any one of the two expressions</p> <p>Either <math>1 \pm 2x</math> or <math>1 \pm 4x</math> from either first or second expansions respectively</p> <p>Ignoring <math>-\frac{3}{2}</math> and <math>\frac{1}{2}</math>, any one correct {.....} expansion.</p> <p>Both {.....} correct.</p> <p>M1</p> <p>dM1;</p> <p>A1</p> <p>A1</p> <p>A1; A1</p> <p><b>[6]</b></p> <p><b>9 marks</b></p>

**Beware:** In part (a) take care to spot that  $A = -\frac{3}{2}$  and  $B = \frac{1}{2}$  are the right way around.

**Beware:** In ePEN, make sure you aware the marks correctly in part (a). The first A1 is for  $A = -\frac{3}{2}$  and the second A1 is for  $B = \frac{1}{2}$ .

**Beware:** If a candidate uses a method of long division please escalate this to you team leader.

Question Number	Scheme	Marks
<b>Aliter</b> <b>2. (b)</b> <b>Way 2</b>	$f(x) = (3x - 1)(1 - 2x)^{-2}$ $= (3x - 1) \times \left( 1 + (-2)(-2x) + \frac{(-2)(-3)}{2!}(-2x)^2 + \frac{(-2)(-3)(-4)}{3!}(-2x)^3 + \dots \right)$ $= (3x - 1)(1 + 4x + 12x^2 + 32x^3 + \dots)$ $= \underline{3x + 12x^2 + 36x^3 - 1 - 4x - 12x^2 - 32x^3 + \dots}$ $= -1 - x + 0x^2 + 4x^3$	Moving power to top M1 Ignoring $(3x - 1)$ , correct $1 \pm 4x$ ; dM1; (.....) expansion A1 <u>Correct expansion</u> A1 $-1 - x$ ; $(0x^2) + 4x^3$ A1; A1 <b>[6]</b>
<b>Aliter</b> <b>2. (b)</b> <b>Way 3</b>	Maclaurin expansion $f(x) = -\frac{3}{2}(1 - 2x)^{-1} + \frac{1}{2}(1 - 2x)^{-2}$ $f'(x) = -3(1 - 2x)^{-2} + 2(1 - 2x)^{-3}$ $f''(x) = -12(1 - 2x)^{-3} + 12(1 - 2x)^{-4}$ $f'''(x) = -72(1 - 2x)^{-4} + 96(1 - 2x)^{-5}$ $\therefore f(0) = -1, f'(0) = -1, f''(0) = 0 \text{ and } f'''(0) = 24$ $\text{gives } f(x) = -1 - x + 0x^2 + 4x^3 + \dots$	Bringing both powers to top M1 Differentiates to give $a(1 - 2x)^{-2} \pm b(1 - 2x)^{-3}$ ; M1; $-3(1 - 2x)^{-2} + 2(1 - 2x)^{-3}$ A1 oe Correct $f''(x)$ and $f'''(x)$ A1 $-1 - x$ ; $(0x^2) + 4x^3$ A1; A1 <b>[6]</b>

Question Number	Scheme	Marks
<p><b>Aliter</b></p> <p>2. (b)</p> <p><b>Way 4</b></p>	$f(x) = -3(2 - 4x)^{-1} + \frac{1}{2}(1 - 2x)^{-2}$ $= -3 \left\{ \begin{aligned} &(2)^{-1} + (-1)(2)^{-2}(-4x); + \frac{(-1)(-2)}{2!} (2)^{-3}(-4x)^2 \\ &+ \frac{(-1)(-2)(-3)}{3!} (2)^{-4}(-4x)^3 + \dots \end{aligned} \right\}$ $+ \frac{1}{2} \left\{ \begin{aligned} &1 + (-2)(-2x); + \frac{(-2)(-3)}{2!} (-2x)^2 + \frac{(-2)(-3)(-4)}{3!} (-2x)^3 + \dots \end{aligned} \right\}$ $= -3 \left\{ \frac{1}{2} + x + 2x^2 + 4x^3 + \dots \right\} + \frac{1}{2} \left\{ 1 + 4x + 12x^2 + 32x^3 + \dots \right\}$ $= -1 - x; + 0x^2 + 4x^3$	<p>Moving powers to top on any one of the two expressions M1</p> <p>Either <math>\frac{1}{2} \pm x</math> or <math>1 \pm 4x</math> from either first or second expansions respectively dM1;</p> <p>Ignoring <math>-3</math> and <math>\frac{1}{2}</math>, any one correct {.....} expansion. A1</p> <p>Both {.....} correct. A1</p> <p><math>-1 - x; (0x^2) + 4x^3</math> A1; A1</p> <p style="text-align: right;"><b>[6]</b></p>

Question Number	Scheme	Marks
3. (a)	<p>Area Shaded = <math>\int_0^{2\pi} 3 \sin\left(\frac{x}{2}\right) dx</math></p> $= \left[ \frac{-3 \cos\left(\frac{x}{2}\right)}{\frac{1}{2}} \right]_0^{2\pi}$ $= [-6 \cos\left(\frac{x}{2}\right)]_0^{2\pi}$ $= [-6(-1)] - [-6(1)] = 6 + 6 = \underline{12}$ <p>(Answer of 12 with no working scores M0A0A0.)</p>	<p>Integrating <math>3 \sin\left(\frac{x}{2}\right)</math> to give <math>k \cos\left(\frac{x}{2}\right)</math> with <math>k \neq 1</math>. Ignore limits.</p> <p>M1</p> <p>-6 cos(<math>\frac{x}{2}</math>) or <math>\frac{-3}{\frac{1}{2}} \cos\left(\frac{x}{2}\right)</math> A1 oe.</p> <p><u>12</u> A1 cao</p> <p>[3]</p>
(b)	<p>Volume = <math>\pi \int_0^{2\pi} \left(3 \sin\left(\frac{x}{2}\right)\right)^2 dx = 9\pi \int_0^{2\pi} \sin^2\left(\frac{x}{2}\right) dx</math></p> <p>[NB: <math>\cos 2x = \pm 1 \pm 2 \sin^2 x</math> gives <math>\sin^2 x = \frac{1 - \cos 2x}{2}</math>]          [NB: <math>\cos x = \pm 1 \pm 2 \sin^2\left(\frac{x}{2}\right)</math> gives <math>\sin^2\left(\frac{x}{2}\right) = \frac{1 - \cos x}{2}</math>]</p> <p><math>\therefore</math> Volume = <math>9(\pi) \int_0^{2\pi} \left(\frac{1 - \cos x}{2}\right) dx</math></p> $= \frac{9(\pi)}{2} \int_0^{2\pi} (1 - \cos x) dx$ $= \frac{9(\pi)}{2} [x - \sin x]_0^{2\pi}$ $= \frac{9\pi}{2} [(2\pi - 0) - (0 - 0)]$ $= \frac{9\pi}{2} (2\pi) = \underline{9\pi^2} \text{ or } \underline{88.8264\dots}$	<p>Use of <math>V = \pi \int y^2 dx</math>. Can be implied. Ignore limits.</p> <p>M1</p> <p>Consideration of the Half Angle Formula for <math>\sin^2\left(\frac{x}{2}\right)</math> or the Double Angle Formula for <math>\sin^2 x</math></p> <p>M1*</p> <p>Correct expression for Volume Ignore limits and <math>\pi</math>.</p> <p>A1</p> <p>Integrating to give <math>\pm ax \pm b \sin x</math>; Correct integration <u><math>k - k \cos x \rightarrow kx - k \sin x</math></u></p> <p>depM1* ; A1</p> <p>Use of limits to give either <math>9\pi^2</math> or awrt 88.8 Solution must be completely correct. No flukes allowed.</p> <p>A1 cso</p> <p>[6]</p>
		<b>9 marks</b>



**Question 3**

**Note:**  $\pi$  is not needed for the middle four marks of question 3(b).

**Beware:** Owing to the symmetry of the curve between  $x = 0$  and  $x = 2\pi$  candidates can find:

- Area =  $2 \int_0^{\pi} 3 \sin\left(\frac{x}{2}\right) dx$  in part (a).

- Volume =  $2\pi \int_0^{\pi} \left(3 \sin\left(\frac{x}{2}\right)\right)^2 dx$

**Beware:** If a candidate gives the correct answer to part (b) with no working please escalate this response up to your team leader.

Question Number	Scheme	Marks
<p>4. (a)</p>	$x = \sin t, \quad y = \sin\left(t + \frac{\pi}{6}\right)$ $\frac{dx}{dt} = \cos t, \quad \frac{dy}{dt} = \cos\left(t + \frac{\pi}{6}\right)$ <p>When <math>t = \frac{\pi}{6}</math>,</p> $\frac{dy}{dx} = \frac{\cos\left(\frac{\pi}{6} + \frac{\pi}{6}\right)}{\cos\left(\frac{\pi}{6}\right)} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58$ <p>When <math>t = \frac{\pi}{6}</math>, <math>x = \frac{1}{2}</math>, <math>y = \frac{\sqrt{3}}{2}</math></p> <p><u>T: <math>y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(x - \frac{1}{2}\right)</math></u></p> <p>or <math>\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(\frac{1}{2}\right) + c \Rightarrow c = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6} = \frac{\sqrt{3}}{3}</math></p> <p>or T: <math>\left[ y = \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3} \right]</math></p>	<p>Attempt to differentiate both x and y wrt t to give two terms in cos</p> <p>Correct <math>\frac{dx}{dt}</math> and <math>\frac{dy}{dt}</math></p> <p>Divides in correct way and substitutes for t to give any of the four underlined oe:</p> <p>Ignore the double negative if candidate has differentiated <math>\sin \rightarrow -\cos</math></p> <p>The point <math>\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)</math> or <math>\left(\frac{1}{2}, \text{awrt } 0.87\right)</math></p> <p>Finding an equation of a tangent with their point and their tangent gradient or finds c and uses <math>y = (\text{their gradient})x + "c"</math>.</p> <p>Correct <u>EXACT</u> equation of <u>tangent</u> oe.</p> <p>[6]</p>
<p>(b)</p>	$y = \sin\left(t + \frac{\pi}{6}\right) = \sin t \cos \frac{\pi}{6} + \cos t \sin \frac{\pi}{6}$ <p>Nb: <math>\sin^2 t + \cos^2 t \equiv 1 \Rightarrow \cos^2 t \equiv 1 - \sin^2 t</math></p> <p><math>\therefore x = \sin t</math> gives <math>\cos t = \sqrt{1 - x^2}</math></p> <p><math>\therefore y = \frac{\sqrt{3}}{2} \sin t + \frac{1}{2} \cos t</math></p> <p>gives <math>y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{1 - x^2}</math> <b>AG</b></p>	<p>Use of compound angle formula for sine.</p> <p>Use of trig identity to find <math>\cos t</math> in terms of x or <math>\cos^2 t</math> in terms of x.</p> <p>Substitutes for <math>\sin t</math>, <math>\cos \frac{\pi}{6}</math>, <math>\cos t</math> and <math>\sin \frac{\pi}{6}</math> to give y in terms of x.</p> <p>[3]</p>
		<b>9 marks</b>

Question Number	Scheme	Marks
<p><b>Aliter</b></p> <p><b>4. (a)</b></p> <p><b>Way 2</b></p>	<p> <math>x = \sin t, \quad y = \sin\left(t + \frac{\pi}{6}\right) = \sin t \cos \frac{\pi}{6} + \cos t \sin \frac{\pi}{6}</math> </p> <p> <math>\frac{dx}{dt} = \cos t, \quad \frac{dy}{dt} = \cos t \cos \frac{\pi}{6} - \sin t \sin \frac{\pi}{6}</math> </p> <p>           When <math>t = \frac{\pi}{6}, \quad \frac{dy}{dx} = \frac{\cos \frac{\pi}{6} \cos \frac{\pi}{6} - \sin \frac{\pi}{6} \sin \frac{\pi}{6}}{\cos\left(\frac{\pi}{6}\right)}</math> </p> <p> <math>= \frac{\frac{3}{4} - \frac{1}{4}}{\frac{\sqrt{3}}{2}} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58</math> </p> <p>           When <math>t = \frac{\pi}{6}, \quad x = \frac{1}{2}, \quad y = \frac{\sqrt{3}}{2}</math> </p> <p> <b>T:</b> <math>y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(x - \frac{1}{2}\right)</math> </p> <p>           or <math>\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(\frac{1}{2}\right) + c \Rightarrow c = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6} = \frac{\sqrt{3}}{3}</math> </p> <p>           or <b>T:</b> <math>\left[ y = \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3} \right]</math> </p>	<p>(Do not give this for part (b))</p> <p>Attempt to differentiate x and y wrt t to give <math>\frac{dx}{dt}</math> in terms of cos and <math>\frac{dy}{dt}</math> in the form <math>\pm a \cos t \pm b \sin t</math></p> <p>M1</p> <p>Correct <math>\frac{dx}{dt}</math> and <math>\frac{dy}{dt}</math></p> <p>A1</p> <p>Divides in correct way and substitutes for t to give any of the four underlined oe:</p> <p>A1</p> <p>The point <math>\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)</math></p> <p>or <math>\left(\frac{1}{2}, \text{awrt } 0.87\right)</math></p> <p>B1</p> <p>Finding an equation of a tangent with their point and their tangent gradient or finds c and uses <math>y = (\text{their gradient})x + "c"</math>.</p> <p>Correct EXACT equation of <u>tangent</u> oe.</p> <p>dM1</p> <p>A1 oe</p> <p>[6]</p>

Question Number	Scheme	Marks
<p><b>Aliter</b></p> <p><b>4. (a)</b></p> <p><b>Way 3</b></p>	$y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{(1-x^2)}$ $\frac{dy}{dx} = \frac{\sqrt{3}}{2} + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(1-x^2)^{-\frac{1}{2}}(-2x)$ $\frac{dy}{dx} = \frac{\sqrt{3}}{2} + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(1-(0.5)^2)^{-\frac{1}{2}}(-2(0.5)) = \frac{1}{\sqrt{3}}$ <p>When <math>t = \frac{\pi}{6}</math>, <math>x = \frac{1}{2}</math>, <math>y = \frac{\sqrt{3}}{2}</math></p> <p><b>T:</b> <math>y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}(x - \frac{1}{2})</math></p> <p>or <math>\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}(\frac{1}{2}) + c \Rightarrow c = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6} = \frac{\sqrt{3}}{3}</math></p> <p>or <b>T:</b> <math>\left[ y = \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3} \right]</math></p>	<p>Attempt to differentiate two terms using the chain rule for the second term. Correct <math>\frac{dy}{dx}</math></p> <p>Correct substitution of <math>x = \frac{1}{2}</math> into a correct <math>\frac{dy}{dx}</math></p> <p>The point <math>\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)</math> or <math>\left(\frac{1}{2}, \text{awrt } 0.87\right)</math></p> <p>Finding an equation of a tangent with their point and their tangent gradient or finds c and uses <math>y = (\text{their gradient})x + "c"</math>. Correct <u>EXACT</u> equation of <u>tangent</u> oe.</p> <p>M1 A1 A1 B1 dM1 A1 oe</p>
<p><b>Aliter</b></p> <p><b>4. (b)</b></p> <p><b>Way 2</b></p>	$x = \sin t \text{ gives } y = \frac{\sqrt{3}}{2} \sin t + \frac{1}{2}\sqrt{(1-\sin^2 t)}$ <p>Nb: <math>\sin^2 t + \cos^2 t \equiv 1 \Rightarrow \cos^2 t \equiv 1 - \sin^2 t</math></p> $\cos t = \sqrt{(1-\sin^2 t)}$ <p>gives <math>y = \frac{\sqrt{3}}{2} \sin t + \frac{1}{2} \cos t</math></p> <p>Hence <math>y = \sin t \cos \frac{\pi}{6} + \cos t \sin \frac{\pi}{6} = \sin\left(t + \frac{\pi}{6}\right)</math></p>	<p>Substitutes <math>x = \sin t</math> into the equation give in y.</p> <p>Use of trig identity to deduce that <math>\cos t = \sqrt{(1-\sin^2 t)}</math>.</p> <p>Using the compound angle formula to prove <math>y = \sin\left(t + \frac{\pi}{6}\right)</math></p> <p>M1 M1 A1 cso</p>
		<b>[6]</b>
		<b>[3]</b>
		<b>9 marks</b>

Question Number	Scheme	Marks
5. (a)	<p>Equating <b>i</b>; <math>0 = 6 + \lambda \Rightarrow \lambda = -6</math></p> <p>Using <math>\lambda = -6</math> and</p> <p>equating <b>j</b>; <math>a = 19 + 4(-6) = -5</math></p> <p>equating <b>k</b>; <math>b = -1 - 2(-6) = 11</math></p> <p>With no working...            ... only one of a or b stated correctly gains the first 2 marks.            ... both a and b stated correctly gains 3 marks.</p>	<p><math>\lambda = -6</math> Can be implied B1 <math>\Rightarrow</math> d</p> <p>For inserting <b>their stated</b> <math>\lambda</math> into either a correct <b>j</b> or <b>k</b> component Can be implied. M1 <math>\Rightarrow</math> d</p> <p><math>a = -5</math> and <math>b = 11</math> A1</p> <p>[3]</p>
(b)	<p><math>\overline{OP} = (6 + \lambda)\mathbf{i} + (19 + 4\lambda)\mathbf{j} + (-1 - 2\lambda)\mathbf{k}</math></p> <p>direction vector or <math>l_1 = \mathbf{d} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}</math></p> <p><math>\overline{OP} \perp l_1 \Rightarrow \overline{OP} \cdot \mathbf{d} = 0</math></p> <p>ie. <math>\begin{pmatrix} 6 + \lambda \\ 19 + 4\lambda \\ -1 - 2\lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = 0</math> (or <u><math>x + 4y - 2z = 0</math></u>)</p> <p><math>\therefore 6 + \lambda + 4(19 + 4\lambda) - 2(-1 - 2\lambda) = 0</math></p> <p><math>6 + \lambda + 76 + 16\lambda + 2 + 4\lambda = 0</math></p> <p><math>21\lambda + 84 = 0 \Rightarrow \lambda = -4</math></p> <p><math>\overline{OP} = (6 - 4)\mathbf{i} + (19 + 4(-4))\mathbf{j} + (-1 - 2(-4))\mathbf{k}</math></p> <p><math>\overline{OP} = 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}</math></p>	<p>Allow <u>this statement</u> for M1 if <math>\overline{OP}</math> and <math>\mathbf{d}</math> are defined as above.</p> <p>Allow either of these two <u>underlined statements</u> M1</p> <p>Correct equation A1 oe</p> <p>Attempt to solve the equation in <math>\lambda</math> dM1</p> <p><math>\lambda = -4</math> A1</p> <p>Substitutes their <math>\lambda</math> into an expression for <math>\overline{OP}</math> M1</p> <p><math>2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}</math> or P(2, 3, 7) A1</p> <p>[6]</p>

**Note:** A similar method may be used by using  $\overline{OP} = (0 + \lambda)\mathbf{i} + (-5 + 4\lambda)\mathbf{j} + (11 - 2\lambda)\mathbf{k}$  and  $\mathbf{d} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$   
 $\overline{OP} \cdot \mathbf{d} = 0$  yields  $6 + \lambda + 4(-5 + 4\lambda) - 2(11 - 2\lambda) = 0$   
 This simplifies to  $21\lambda - 42 = 0 \Rightarrow \lambda = 2$ .  
 $\overline{OP} = (0 + 2)\mathbf{i} + (-5 + 4(2))\mathbf{j} + (11 - 2(2))\mathbf{k}$   
 $\overline{OP} = 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$

Question Number	Scheme	Marks
<b>Aliter</b> (b) <b>Way 2</b>	$\overline{OP} = (6 + \lambda)\mathbf{i} + (19 + 4\lambda)\mathbf{j} + (-1 - 2\lambda)\mathbf{k}$ $\overline{AP} = (6 + \lambda - 0)\mathbf{i} + (19 + 4\lambda + 5)\mathbf{j} + (-1 - 2\lambda - 11)\mathbf{k}$ <p>direction vector or <math>l_1 = \mathbf{d} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}</math></p> $\overline{AP} \perp \overline{OP} \Rightarrow \underline{\overline{AP} \cdot \overline{OP} = 0}$ <p>ie. <math display="block">\begin{pmatrix} 6 + \lambda \\ 24 + 4\lambda \\ -12 - 2\lambda \end{pmatrix} \cdot \begin{pmatrix} 6 + \lambda \\ 19 + 4\lambda \\ -1 - 2\lambda \end{pmatrix} = 0</math></p> $\therefore (6 + \lambda)(6 + \lambda) + (24 + 4\lambda)(19 + 4\lambda) + (-12 - 2\lambda)(-1 - 2\lambda) = 0$ $36 + 12\lambda + \lambda^2 + 456 + 96\lambda + 76\lambda + 16\lambda^2 + 12 + 24\lambda + 2\lambda + 4\lambda^2 = 0$ $21\lambda^2 + 210\lambda + 504 = 0$ $\lambda^2 + 10\lambda + 24 = 0 \Rightarrow (\lambda = -6) \quad \underline{\lambda = -4}$ $\overline{OP} = (6 - 4)\mathbf{i} + (19 + 4(-4))\mathbf{j} + (-1 - 2(-4))\mathbf{k}$ $\overline{OP} = 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$	<p>Allow <u>this statement</u> for M1 if <math>\overline{AP}</math> and <math>\overline{OP}</math> are defined as above.</p> <p><u>underlined statement</u> M1</p> <p>Correct equation A1 oe</p> <p>Attempt to solve the equation in <math>\lambda</math> dM1</p> <p><math>\lambda = -4</math> A1</p> <p>Substitutes their <math>\lambda</math> into an expression for <math>\overline{OP}</math> M1</p> <p><math>2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}</math> or P(2, 3, 7) A1</p> <p>[6]</p>

**Note:** A similar method to way 2 may be used by using  $\overline{OP} = (5 + \lambda)\mathbf{i} + (15 + 4\lambda)\mathbf{j} + (1 - 2\lambda)\mathbf{k}$

and  $\overline{AP} = (5 + \lambda - 0)\mathbf{i} + (15 + 4\lambda + 5)\mathbf{j} + (1 - 2\lambda - 11)\mathbf{k}$

$\overline{AP} \cdot \overline{OP} = 0$  yields  $(5 + \lambda)(5 + \lambda) + (20 + 4\lambda)(15 + 4\lambda) + (-10 - 2\lambda)(1 - 2\lambda) = 0$

This simplifies to  $21\lambda^2 + 168\lambda + 315 = 0$ .  $\lambda^2 + 8\lambda + 15 = 0 \Rightarrow (\lambda = -5) \quad \underline{\lambda = -3}$

$\overline{OP} = (5 - 3)\mathbf{i} + (15 + 4(-3))\mathbf{j} + (1 - 2(-3))\mathbf{k}$

$\overline{OP} = 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$

Question Number	Scheme	Marks
<p>5. (c)</p> <p><b>Aliter</b></p> <p>5. (c)</p> <p>Way 2</p>	<p> <math>\overline{OP} = 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}</math>  <math>\overline{OA} = 0\mathbf{i} - 5\mathbf{j} + 11\mathbf{k}</math> and <math>\overline{OB} = 5\mathbf{i} + 15\mathbf{j} + \mathbf{k}</math>  <math>\overline{AP} = \pm(2\mathbf{i} + 8\mathbf{j} - 4\mathbf{k})</math>, <math>\overline{PB} = \pm(3\mathbf{i} + 12\mathbf{j} - 6\mathbf{k})</math>  <math>\overline{AB} = \pm(5\mathbf{i} + 20\mathbf{j} - 10\mathbf{k})</math> </p> <p>           As <math>\overline{AP} = \frac{2}{3}(3\mathbf{i} + 12\mathbf{j} - 6\mathbf{k}) = \frac{2}{3}\overline{PB}</math>            or <math>\overline{AB} = \frac{5}{2}(2\mathbf{i} + 8\mathbf{j} - 4\mathbf{k}) = \frac{5}{2}\overline{AP}</math>            or <math>\overline{AB} = \frac{5}{3}(3\mathbf{i} + 12\mathbf{j} - 6\mathbf{k}) = \frac{5}{3}\overline{PB}</math>            or <math>\overline{PB} = \frac{3}{2}(2\mathbf{i} + 8\mathbf{j} - 4\mathbf{k}) = \frac{3}{2}\overline{AP}</math>            or <math>\overline{AP} = \frac{2}{5}(5\mathbf{i} + 20\mathbf{j} - 10\mathbf{k}) = \frac{2}{5}\overline{AB}</math>            or <math>\overline{PB} = \frac{3}{5}(5\mathbf{i} + 20\mathbf{j} - 10\mathbf{k}) = \frac{3}{5}\overline{AB}</math> etc...         </p> <p>alternatively candidates could say for example that</p> <p><math>\overline{AP} = 2(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})</math>    <math>\overline{PB} = 3(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})</math></p> <p>then <u>the points A, P and B are collinear.</u></p> <p><math>\therefore \overline{AP} : \overline{PB} = 2 : 3</math></p> <p>At B; <u><math>5 = 6 + \lambda</math></u>, <u><math>15 = 19 + 4\lambda</math></u> or <u><math>1 = -1 - 2\lambda</math></u>            or at B; <math>\lambda = -1</math></p> <p>gives <math>\lambda = -1</math> for all three equations.            or when <math>\lambda = -1</math>, this gives <math>\mathbf{r} = 5\mathbf{i} + 15\mathbf{j} + \mathbf{k}</math></p> <p><u>Hence B lies on <math>l_1</math>.</u> As stated in the question both A and P lie on <math>l_1</math>. <math>\therefore</math> <u>A, P and B are collinear.</u></p> <p><math>\therefore \overline{AP} : \overline{PB} = 2 : 3</math></p>	<p>Subtracting vectors to find any two of <math>\overline{AP}</math>, <math>\overline{PB}</math> or <math>\overline{AB}</math>; and both are correctly ft using candidate's <math>\overline{OA}</math> and <math>\overline{OP}</math> found in parts (a) and (b) respectively.</p> <p>M1; A1 <math>\sqrt{\pm}</math></p> <p>A, P and B are collinear Completely correct proof. A1</p> <p>2:3 or <math>1 : \frac{3}{2}</math> or <math>\sqrt{84} : \sqrt{189}</math> aef B1 oe allow SC <math>\frac{2}{3}</math> [4]</p> <p>Writing down any of the three <u>underlined equations.</u> M1</p> <p><math>\lambda = -1</math> for all three equations or <math>\lambda = -1</math> gives <math>\mathbf{r} = 5\mathbf{i} + 15\mathbf{j} + \mathbf{k}</math> A1</p> <p><u>Must state B lies on <math>l_1</math></u> <math>\Rightarrow</math> A, P and B are collinear A1</p> <p>2:3 or aef B1 oe [4]</p> <p><b>13 marks</b></p>

**Beware** of candidates who will try to fudge that one vector is multiple of another for the final A mark in part (c).

Question Number	Scheme	Marks																		
6. (a)	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding: 2px 10px;">x</td> <td style="padding: 2px 10px;">1</td> <td style="padding: 2px 10px;">1.5</td> <td style="padding: 2px 10px;">2</td> <td style="padding: 2px 10px;">2.5</td> <td style="padding: 2px 10px;">3</td> </tr> <tr> <td style="padding: 2px 10px;">y</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">0.5 ln 1.5</td> <td style="padding: 2px 10px;">ln 2</td> <td style="padding: 2px 10px;">1.5 ln 2.5</td> <td style="padding: 2px 10px;">2 ln 3</td> </tr> <tr> <td style="padding: 2px 10px;">or y</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">0.2027325541...</td> <td style="padding: 2px 10px;">ln2</td> <td style="padding: 2px 10px;">1.374436098...</td> <td style="padding: 2px 10px;">2 ln 3</td> </tr> </table> <p style="text-align: right; margin-right: 100px;">Either 0.5 ln 1.5 and 1.5 ln 2.5 or awrt 0.20 and 1.37 (or mixture of decimals and ln's)</p>	x	1	1.5	2	2.5	3	y	0	0.5 ln 1.5	ln 2	1.5 ln 2.5	2 ln 3	or y	0	0.2027325541...	ln2	1.374436098...	2 ln 3	B1 [1]
x	1	1.5	2	2.5	3															
y	0	0.5 ln 1.5	ln 2	1.5 ln 2.5	2 ln 3															
or y	0	0.2027325541...	ln2	1.374436098...	2 ln 3															
(b)(i)	$I_1 \approx \frac{1}{2} \times 1 \times \{0 + 2(\ln 2) + 2\ln 3\}$ $= \frac{1}{2} \times 3.583518938... = 1.791759... = 1.792 \text{ (4sf)}$	<p style="text-align: center;"><u>For structure of trapezium rule</u> {.....};</p> M1;  1.792 A1 cao																		
(ii)	$I_2 \approx \frac{1}{2} \times 0.5 \times \{0 + 2(0.5\ln 1.5 + \ln 2 + 1.5\ln 2.5) + 2\ln 3\}$ $= \frac{1}{4} \times 6.737856242... = 1.684464...$	<p style="text-align: center;">Outside brackets <math>\frac{1}{2} \times 0.5</math></p> <p style="text-align: center;"><u>For structure of trapezium rule</u> {.....};</p> M1 $\sqrt{\quad}$  awrt 1.684 A1																		
(c)	<p>With increasing ordinates, <u>the line segments at the top of the trapezia are closer to the curve.</u></p>	<p><u>Reason</u> or an appropriate diagram elaborating the correct reason.</p> B1 [1]																		

**Beware:** In part (b) candidate can add up the individual trapezia:

(b)(i)  $I_1 \approx \frac{1}{2}(0 + \ln 2) + \frac{1}{2}(\ln 2 + \ln 3)$

(ii)  $I_2 \approx \frac{1}{2} \cdot \frac{1}{2}(0 + 0.5\ln 1.5) + \frac{1}{2} \cdot \frac{1}{2}(0.5\ln 1.5 + \ln 2) + \frac{1}{2} \cdot \frac{1}{2}(\ln 2 + 1.5\ln 2.5) + \frac{1}{2} \cdot \frac{1}{2}(1.5\ln 2.5 + 2\ln 3)$



Question Number	Scheme	Marks
6. (d)	$\left\{ \begin{array}{l} u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = x - 1 \Rightarrow v = \frac{x^2}{2} - x \end{array} \right\}$ $I = \left( \frac{x^2}{2} - x \right) \ln x - \int \frac{1}{x} \left( \frac{x^2}{2} - x \right) dx$ $= \left( \frac{x^2}{2} - x \right) \ln x - \int \left( \frac{x}{2} - 1 \right) dx$ $= \left( \frac{x^2}{2} - x \right) \ln x - \left( \frac{x^2}{4} - x \right) (+c)$ $\therefore I = \left[ \left( \frac{x^2}{2} - x \right) \ln x - \frac{x^2}{4} + x \right]_1^3$ $= \left( \frac{3}{2} \ln 3 - \frac{9}{4} + 3 \right) - \left( -\frac{1}{2} \ln 1 - \frac{1}{4} + 1 \right)$ $= \frac{3}{2} \ln 3 + \frac{3}{4} + 0 - \frac{3}{4} = \frac{3}{2} \ln 3 \quad \mathbf{AG}$	<p>Use of 'integration by parts' formula in the correct direction M1</p> <p>Correct expression A1</p> <p>An attempt to multiply at least one term through by <math>\frac{1}{x}</math> and an attempt to ...</p> <p>... integrate; M1;</p> <p><u>correct integration</u> A1</p> <p>Substitutes limits of 3 and 1 and subtracts. ddM1</p> <p><math>\frac{3}{2} \ln 3</math> A1 cso</p> <p>[6]</p>
<p><b>Aliter</b></p> <p>6. (d)</p> <p><b>Way 2</b></p>	$\int (x - 1) \ln x \, dx = \int x \ln x \, dx - \int \ln x \, dx$ $\int x \ln x \, dx = \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \cdot \left( \frac{1}{x} \right) dx$ $= \frac{x^2}{2} \ln x - \frac{x^2}{4} (+c)$ $\int \ln x \, dx = x \ln x - \int x \cdot \left( \frac{1}{x} \right) dx$ $= x \ln x - x (+c)$ $\therefore \int_1^3 (x - 1) \ln x \, dx = \left( \frac{9}{2} \ln 3 - 2 \right) - (3 \ln 3 - 2) = \frac{3}{2} \ln 3 \quad \mathbf{AG}$	<p>Correct application of 'by parts' M1</p> <p>Correct integration A1</p> <p>Correct application of 'by parts' M1</p> <p>Correct integration A1</p> <p>Substitutes limits of 3 and 1 into both integrands and subtracts. ddM1</p> <p><math>\frac{3}{2} \ln 3</math> A1 cso</p> <p>[6]</p>

Question Number	Scheme	Marks
<p><b>Aliter</b></p> <p><b>6. (d)</b></p> <p><b>Way 3</b></p>	$\left\{ \begin{array}{l} u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = (x-1) \Rightarrow v = \frac{(x-1)^2}{2} \end{array} \right\}$ <p>Use of 'integration by parts' formula in the correct direction</p> $I = \frac{(x-1)^2}{2} \ln x - \int \frac{(x-1)^2}{2x} dx$ <p>Correct expression</p> $= \frac{(x-1)^2}{2} \ln x - \int \frac{x^2 - 2x + 1}{2x} dx$ $= \frac{(x-1)^2}{2} \ln x - \int \left( \frac{1}{2}x - 1 + \frac{1}{2x} \right) dx$ $= \frac{(x-1)^2}{2} \ln x - \left( \frac{x^2}{4} - x + \frac{1}{2} \ln x \right) (+c)$ <p>Candidate multiplies out numerator to obtain three terms...</p> <p>... multiplies at least one term through by <math>\frac{1}{x}</math> and then attempts to ...</p> <p>... integrate the result;</p> <p><u>correct integration</u></p> $\therefore I = \left[ \frac{(x-1)^2}{2} \ln x - \frac{x^2}{4} + x - \frac{1}{2} \ln x \right]_1^3$ <p>Substitutes limits of 3 and 1 and subtracts.</p> $= (2 \ln 3 - \frac{9}{4} + 3 - \frac{1}{2} \ln 3) - (0 - \frac{1}{4} + 1 - 0)$ $= 2 \ln 3 - \frac{1}{2} \ln 3 + \frac{3}{4} + \frac{1}{4} - 1 = \frac{3}{2} \ln 3 \quad \mathbf{AG}$	<p>M1</p> <p>A1</p> <p>M1;</p> <p>A1</p> <p>ddM1</p> <p>A1 cso</p> <p><b>[6]</b></p>

**Beware:**  $\int \frac{1}{2x} dx$  can also integrate to  $\frac{1}{2} \ln 2x$

**Beware:** If you are marking using WAY 2 please make sure that you allocate the marks in the order they appear on the mark scheme. For example if a candidate only integrated  $\ln x$  correctly then they would be awarded M0A0M1A1M0A0 on ePEN.

Question Number	Scheme	Marks
<b>Aliter</b> <b>6. (d)</b> <b>Way 4</b>	<p>By substitution  <math>u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x}</math></p> $I = \int (e^u - 1).ue^u du$ $= \int u(e^{2u} - e^u) du$ $= u\left(\frac{1}{2}e^{2u} - e^u\right) - \int \left(\frac{1}{2}e^{2u} - e^u\right) dx$ $= u\left(\frac{1}{2}e^{2u} - e^u\right) - \left(\frac{1}{4}e^{2u} - e^u\right) (+c)$ $\therefore I = \left[ \frac{1}{2}ue^{2u} - ue^u - \frac{1}{4}e^{2u} + e^u \right]_{\ln 1}^{\ln 3}$ $= \left(\frac{9}{2}\ln 3 - 3\ln 3 - \frac{9}{4} + 3\right) - \left(0 - 0 - \frac{1}{4} + 1\right)$ $= \frac{3}{2}\ln 3 + \frac{3}{4} + \frac{1}{4} - 1 = \underline{\underline{\frac{3}{2}\ln 3}} \quad \mathbf{AG}$	<p>Correct expression</p> <p>Use of 'integration by parts' formula in the correct direction</p> <p>Correct expression</p> <p>Attempt <u>to integrate</u>; <u>correct integration</u></p> <p>Substitutes limits of <math>\ln 3</math> and <math>\ln 1</math> and subtracts.</p> <p><math>\frac{3}{2}\ln 3</math></p> <p>[6]</p>
		<b>13 marks</b>

Question Number	Scheme	Marks
7. (a)	<p>From question, <math>\frac{dS}{dt} = 8</math></p> <p><math>S = 6x^2 \Rightarrow \frac{dS}{dx} = 12x</math></p> <p><math>\frac{dx}{dt} = \frac{dS}{dt} \div \frac{dS}{dx} = \frac{8}{12x}; = \frac{2}{3x} \Rightarrow (k = \frac{2}{3})</math></p>	<p><math>\frac{dS}{dt} = 8</math> B1</p> <p><math>\frac{dS}{dx} = 12x</math> B1</p> <p>Candidate's <math>\frac{dS}{dt} \div \frac{dS}{dx}; \frac{8}{12x}</math> M1; <u>A1</u>oe</p> <p>[4]</p>
(b)	<p><math>V = x^3 \Rightarrow \frac{dV}{dx} = 3x^2</math></p> <p><math>\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} = 3x^2 \cdot \left(\frac{2}{3x}\right); = 2x</math></p> <p>As <math>x = V^{\frac{1}{3}}</math>, then <math>\frac{dV}{dt} = 2V^{\frac{1}{3}}</math> <b>AG</b></p>	<p><math>\frac{dV}{dx} = 3x^2</math> B1</p> <p>Candidate's <math>\frac{dV}{dx} \times \frac{dx}{dt}; \lambda x</math> M1; A1 <math>\sqrt{\quad}</math></p> <p>Use of <math>x = V^{\frac{1}{3}}</math>, to give <math>\frac{dV}{dt} = 2V^{\frac{1}{3}}</math> A1</p> <p>[4]</p>
(c)	<p><math>\int \frac{dV}{V^{\frac{1}{3}}} = \int 2 dt</math></p> <p><math>\int V^{-\frac{1}{3}} dV = \int 2 dt</math></p> <p><math>\frac{3}{2} V^{\frac{2}{3}} = 2t (+c)</math></p> <p><math>\frac{3}{2} (8)^{\frac{2}{3}} = 2(0) + c \Rightarrow c = 6</math></p> <p>Hence: <math>\frac{3}{2} V^{\frac{2}{3}} = 2t + 6</math></p> <p><math>\frac{3}{2} (16\sqrt{2})^{\frac{2}{3}} = 2t + 6 \Rightarrow 12 = 2t + 6</math></p> <p>giving <math>t = 3</math>.</p>	<p>Separates the variables with <math>\int \frac{dV}{V^{\frac{1}{3}}}</math> or <math>\int V^{-\frac{1}{3}} dV</math> on one side and <math>\int 2 dt</math> on the other side. integral signs not necessary. B1</p> <p>Attempts to integrate and ... ... must see <math>V^{\frac{2}{3}}</math> and <math>2t</math>; Correct equation with/without <math>+c</math>. M1; A1</p> <p>Use of <math>V = 8</math> and <math>t = 0</math> in a changed equation containing <math>c</math> ; <math>c = 6</math> M1* ; A1</p> <p>Having found their "c" candidate ... ... substitutes <math>V = 16\sqrt{2}</math> into an equation involving <math>V, t</math> and "c". depM1*</p> <p><math>t = 3</math> A1 cao</p> <p>[7]</p>
		<b>15 marks</b>

Question Number	Scheme	Marks
<p><b>Aliter</b></p> <p>7. (b)</p> <p><b>Way 2</b></p>	$x = V^{\frac{1}{3}} \text{ \& } S = 6x^2 \Rightarrow S = 6V^{\frac{2}{3}} \qquad S = 6V^{\frac{2}{3}}$ $\frac{dS}{dV} = 4V^{-\frac{1}{3}} \text{ or } \frac{dV}{dS} = \frac{1}{4}V^{\frac{1}{3}}$ $\frac{dV}{dt} = \frac{dS}{dt} \times \frac{dV}{dS} = 8 \cdot \left( \frac{1}{4V^{\frac{1}{3}}} \right); = \frac{2}{V^{\frac{1}{3}}} = 2V^{-\frac{1}{3}} \text{ \textbf{AG}}$	<p>B1 <math>\sqrt{\quad}</math></p> <p>B1</p> <p>M1; A1</p> <p style="text-align: center;"><b>In ePEN, award Marks for Way 2 in the order they appear on this mark scheme.</b></p> <p style="text-align: right;"><b>[4]</b></p>
<p><b>Aliter</b></p> <p>7. (c)</p> <p><b>Way 2</b></p>	$\int \frac{dV}{2V^{\frac{1}{3}}} = \int 1 dt$ $\frac{1}{2} \int V^{-\frac{1}{3}} dV = \int 1 dt$ $\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)V^{\frac{2}{3}} = t (+c)$ $\frac{3}{4}(8)^{\frac{2}{3}} = (0) + c \Rightarrow c = 3$ <p>Hence: <math>\frac{3}{4}V^{\frac{2}{3}} = t + 3</math></p> $\frac{3}{4}(16\sqrt{2})^{\frac{2}{3}} = t + 3 \Rightarrow 6 = t + 3$ <p>giving <math>t = 3</math>.</p>	<p>Separates the variables with <math>\int \frac{dV}{2V^{\frac{1}{3}}}</math> or <math>\int \frac{1}{2}V^{-\frac{1}{3}}dV</math> oe on one side and <math>\int 1 dt</math> on the other side. integral signs not necessary.</p> <p>Attempts to integrate and ... ... must see <math>V^{\frac{2}{3}}</math> and <math>t</math>; Correct equation with/without <math>+ c</math>.</p> <p>Use of <math>V = 8</math> and <math>t = 0</math> in a changed equation containing <math>c</math>; <math>c = 3</math></p> <p>Having found their "c" candidate ... ... substitutes <math>V = 16\sqrt{2}</math> into an equation involving <math>V</math>, <math>t</math> and "c".</p> <p><math>t = 3</math></p> <p>M1; A1</p> <p>M1*; A1</p> <p>depM1*</p> <p>A1 cao</p> <p style="text-align: right;"><b>[7]</b></p>

**Beware:** On ePEN award the marks in part (c) in the order they appear on the mark scheme.

Question Number	Scheme	Marks
<b>Aliter</b>	<i>similar to way 1.</i>	
(b)	$V = x^3 \Rightarrow \frac{dV}{dx} = 3x^2$	B1
<b>Way 3</b>	$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dS}{dt} \times \frac{dx}{dS} = 3x^2 \cdot 8 \cdot \left(\frac{1}{12x}\right); = 2x$	M1; A1 $\sqrt{\quad}$
	As $x = V^{\frac{1}{3}}$ , then $\frac{dV}{dt} = 2V^{\frac{1}{3}}$ <b>AG</b>	A1
<b>Aliter</b>		[4]
(c)		B1
<b>Way 3</b>		
	$\int \frac{dV}{V^{\frac{1}{3}}} = \int 2 dt$	Separates the variables with $\int \frac{dV}{V^{\frac{1}{3}}}$ or $\int V^{-\frac{1}{3}} dV$ on one side and $\int 2 dt$ on the other side. integral signs not necessary.
	$\int V^{-\frac{1}{3}} dV = \int 2 dt$	
	$V^{\frac{2}{3}} = \frac{4}{3}t (+c)$	Attempts to integrate and ... ... must see $V^{\frac{2}{3}}$ and $\frac{4}{3}t$ ; A1
	$(8)^{\frac{2}{3}} = \frac{4}{3}(0) + c \Rightarrow c = 4$	Use of $V = 8$ and $t = 0$ in a changed equation containing $c$ ; $c = 4$ M1* ; A1
	Hence: $V^{\frac{2}{3}} = \frac{4}{3}t + 4$	
	$(16\sqrt{2})^{\frac{2}{3}} = \frac{4}{3}t + 6 \Rightarrow 8 = \frac{4}{3}t + 4$	Having found their "c" candidate ... ... substitutes $V = 16\sqrt{2}$ into an equation involving $V$ , $t$ and "c". depM1*
	giving $t = 3$ .	$t = 3$ A1 cao [7]

- **Beware** when marking question 7(c). There are a variety of valid ways that a candidate can use to find the constant "c".
- In questions 7(b) and 7(c) there may be "Ways" that I have not listed. Please use the mark scheme as a guide of how the mark the students' responses.
- In 7(c), if a candidate instead tries to solve the differential equation in part (a) escalate the response to your team leader.
- IF YOU ARE UNSURE ON HOW TO APPLY THE MARK SCHEME PLEASE ESCALATE THE RESPONSE UP TO YOUR TEAM LEADER VIA THE REVIEW SYSTEM.
- Note: dM1 denotes a method mark which is dependent upon the award of the previous method mark.  
 ddM1 denotes a method mark which is dependent upon the award of the previous two method marks.  
 depM1\* denotes a method mark which is dependent upon the award of M1\*.