

1. The curve C has the equation $2x + 3y^2 + 3x^2y = 4x^2$.
The point P on the curve has coordinates $(-1, 1)$.

(a) Find the gradient of the curve at P . (5)

(b) Hence find the equation of the normal to C at P , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers. (3)



2. (a) Use integration by parts to find $\int x \sin 3x \, dx$. **(3)**

(b) Using your answer to part (a), find $\int x^2 \cos 3x \, dx$. **(3)**



3. (a) Expand

$$\frac{1}{(2-5x)^2}, \quad |x| < \frac{2}{5}$$

in ascending powers of x , up to and including the term in x^2 , giving each term as a simplified fraction.

(5)

Given that the binomial expansion of $\frac{2+kx}{(2-5x)^2}, \quad |x| < \frac{2}{5}$, is

$$\frac{1}{2} + \frac{7}{4}x + Ax^2 + \dots$$

(b) find the value of the constant k ,

(2)

(c) find the value of the constant A .

(2)



4.

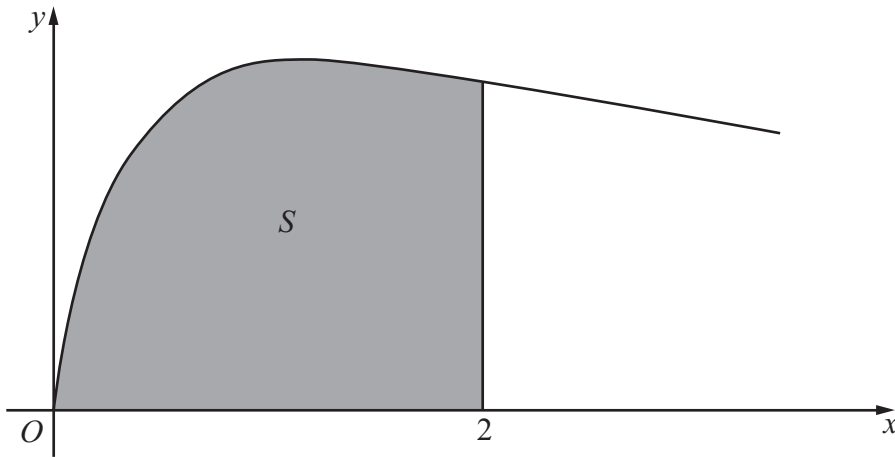


Figure 1

Figure 1 shows the curve with equation

$$y = \sqrt{\left(\frac{2x}{3x^2 + 4}\right)}, \quad x \geq 0$$

The finite region S , shown shaded in Figure 1, is bounded by the curve, the x -axis and the line $x = 2$

The region S is rotated 360° about the x -axis.

Use integration to find the exact value of the volume of the solid generated, giving your answer in the form $k \ln a$, where k and a are constants.

(5)



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Question 4 continued

Handwritten response area consisting of 30 horizontal lines.

(Total 5 marks)

Q4



5.

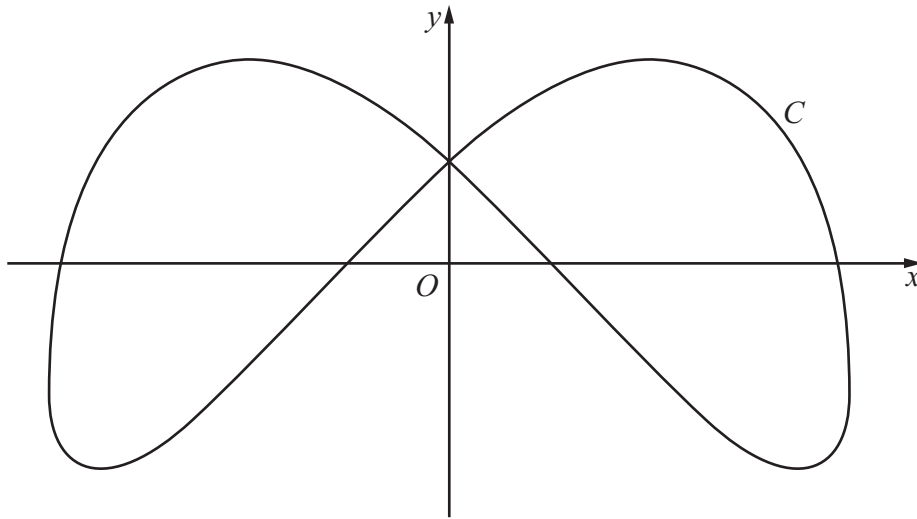


Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 4 \sin\left(t + \frac{\pi}{6}\right), \quad y = 3 \cos 2t, \quad 0 \leq t < 2\pi$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t . (3)

(b) Find the coordinates of all the points on C where $\frac{dy}{dx} = 0$. (5)

6.

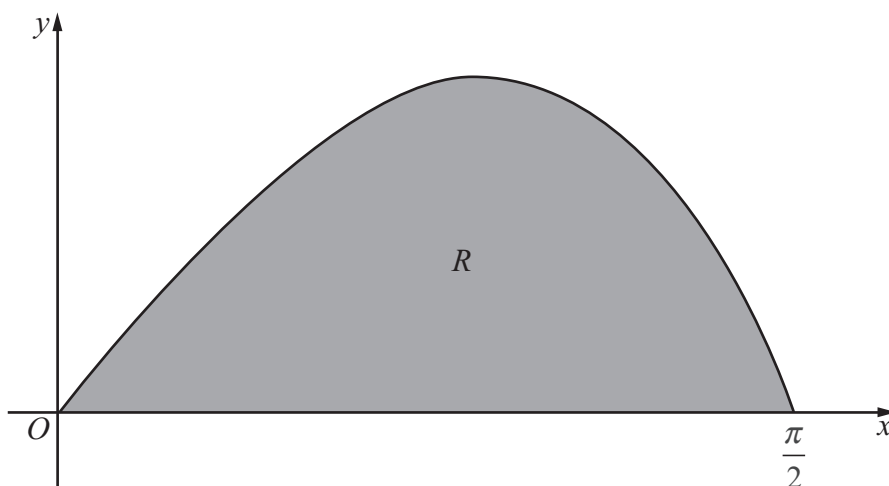


Figure 3

Figure 3 shows a sketch of the curve with equation $y = \frac{2 \sin 2x}{(1 + \cos x)}$, $0 \leq x \leq \frac{\pi}{2}$.

The finite region R , shown shaded in Figure 3, is bounded by the curve and the x -axis.

The table below shows corresponding values of x and y for $y = \frac{2 \sin 2x}{(1 + \cos x)}$.

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y	0		1.17157	1.02280	0

(a) Complete the table above giving the missing value of y to 5 decimal places. (1)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R , giving your answer to 4 decimal places. (3)

(c) Using the substitution $u = 1 + \cos x$, or otherwise, show that

$$\int \frac{2 \sin 2x}{(1 + \cos x)} dx = 4 \ln(1 + \cos x) - 4 \cos x + k$$

where k is a constant. (5)

(d) Hence calculate the error of the estimate in part (b), giving your answer to 2 significant figures. (3)



7. Relative to a fixed origin O , the point A has position vector $(2\mathbf{i} - \mathbf{j} + 5\mathbf{k})$, the point B has position vector $(5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k})$, and the point D has position vector $(-\mathbf{i} + \mathbf{j} + 4\mathbf{k})$.

The line l passes through the points A and B .

- (a) Find the vector \overrightarrow{AB} . (2)
- (b) Find a vector equation for the line l . (2)
- (c) Show that the size of the angle BAD is 109° , to the nearest degree. (4)

The points A , B and D , together with a point C , are the vertices of the parallelogram $ABCD$, where $\overrightarrow{AB} = \overrightarrow{DC}$.

- (d) Find the position vector of C . (2)
- (e) Find the area of the parallelogram $ABCD$, giving your answer to 3 significant figures. (3)
- (f) Find the shortest distance from the point D to the line l , giving your answer to 3 significant figures. (2)



8. (a) Express $\frac{1}{P(5-P)}$ in partial fractions. **(3)**

A team of conservationists is studying the population of meerkats on a nature reserve. The population is modelled by the differential equation

$$\frac{dP}{dt} = \frac{1}{15}P(5 - P), \quad t \geq 0$$

where P , in thousands, is the population of meerkats and t is the time measured in years since the study began.

Given that when $t = 0$, $P = 1$,

- (b) solve the differential equation, giving your answer in the form,

$$P = \frac{a}{b + ce^{-\frac{1}{3}t}}$$

where a , b and c are integers.

(8)

- (c) Hence show that the population cannot exceed 5000

(1)



Question 8 continued

A series of horizontal lines for writing the answer to Question 8.

Q8

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(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

END

