

June 2009  
6665 Core Mathematics C3  
Mark Scheme

Question Number	Scheme	Marks
1. (a)	<p>Iterative formula: <math>x_{n+1} = \frac{2}{(x_n)^2} + 2</math> , <math>x_0 = 2.5</math></p> <p><math>x_1 = \frac{2}{(2.5)^2} + 2</math></p> <p><math>x_1 = 2.32</math></p> <p><math>x_2 = 2.371581451\dots</math></p> <p><math>x_3 = 2.355593575\dots</math></p> <p><math>x_4 = 2.360436923\dots</math></p>	<p>An attempt to substitute <math>x_0 = 2.5</math> into the iterative formula. Can be implied by <math>x_1 = 2.32</math> or <math>2.320</math> Both <math>x_1 = 2.32(0)</math> and <math>x_2 = \text{awrt } 2.372</math> Both <math>x_3 = \text{awrt } 2.356</math> and <math>x_4 = \text{awrt } 2.360</math> or <math>2.36</math></p> <p>M1 A1 A1 cso [3]</p>
(b)	<p>Let <math>f(x) = -x^3 + 2x^2 + 2 = 0</math></p> <p><math>f(2.3585) = 0.00583577\dots</math> <math>f(2.3595) = -0.00142286\dots</math> Sign change (and <math>f(x)</math> is continuous) therefore a root <math>\alpha</math> is such that <math>\alpha \in (2.3585, 2.3595) \Rightarrow \alpha = 2.359</math> (3 dp)</p>	<div style="border: 1px solid black; padding: 2px; margin-bottom: 5px;">Choose suitable interval for <math>x</math>, e.g. <math>[2.3585, 2.3595]</math> or tighter</div> <p>M1 any one value awrt 1 sf or truncated 1 sf dM1</p> <div style="border: 1px solid black; padding: 2px; margin-bottom: 5px;">both values correct, sign change and conclusion</div> <p>A1 [3]</p>
<b>6 marks</b>		

See appendix!

At a minimum, both values must be correct to 1sf or truncated 1sf, candidate states “change of sign, hence root”.

Question Number	Scheme	Marks
2. (a)	$\cos^2 \theta + \sin^2 \theta = 1 \quad (\div \cos^2 \theta)$ $\frac{\cos^2 \theta}{\cos^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}$ $1 + \tan^2 \theta = \sec^2 \theta$ $\tan^2 \theta = \sec^2 \theta - 1 \quad (\text{as required}) \quad \mathbf{AG}$	<p>Dividing <math>\cos^2 \theta + \sin^2 \theta = 1</math> by <math>\cos^2 \theta</math> to give <u>underlined</u> equation. M1</p> <p>Complete proof. No errors seen. (see alternatives in appendix!) A1 cso [2]</p>
(b)	$2 \tan^2 \theta + 4 \sec \theta + \sec^2 \theta = 2, \quad (\text{eqn } *) \quad 0 \leq \theta < 360^\circ$ $2(\sec^2 \theta - 1) + 4 \sec \theta + \sec^2 \theta = 2$ $2 \sec^2 \theta - 2 + 4 \sec \theta + \sec^2 \theta = 2$ $3 \sec^2 \theta + 4 \sec \theta - 4 = 0$ $(\sec \theta + 2)(3 \sec \theta - 2) = 0$ $\sec \theta = -2 \quad \text{or} \quad \sec \theta = \frac{2}{3}$ $\frac{1}{\cos \theta} = -2 \quad \text{or} \quad \frac{1}{\cos \theta} = \frac{2}{3}$ $\underline{\cos \theta = -\frac{1}{2}}; \quad \text{or} \quad \underline{\cos \theta = \frac{3}{2}}$ $\alpha = 120^\circ \quad \text{or} \quad \alpha = \text{no solutions}$ $\theta_1 = \underline{120^\circ}$ $\theta_2 = 240^\circ$ $\theta = \{120^\circ, 240^\circ\}$	<p>Substituting <math>\tan^2 \theta = \sec^2 \theta - 1</math> into eqn * to get a quadratic in <math>\sec \theta</math> only M1</p> <p>Forming a three term “one sided” quadratic expression in <math>\sec \theta</math>. M1</p> <p>Attempt to factorise or solve a quadratic. (See rules for factorising quadratics) M1</p> <p><math>\underline{\cos \theta = -\frac{1}{2}}</math> A1;</p> <p><math>\underline{120^\circ}</math> A1</p> <p><math>\underline{240^\circ}</math> or <math>\theta_2 = 360^\circ - \theta_1</math> when solving using <math>\cos \theta = \dots</math> B1 <math>\sqrt{\quad}</math></p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>Note the final A1 mark has been changed to a B1 mark.</p> </div> <p>[6]</p> <p><b>8 marks</b></p>

**Note:** Please refer to the appendix if candidate offers any extra solutions.

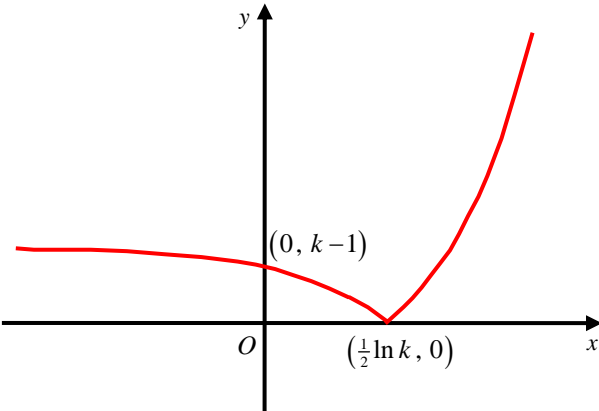
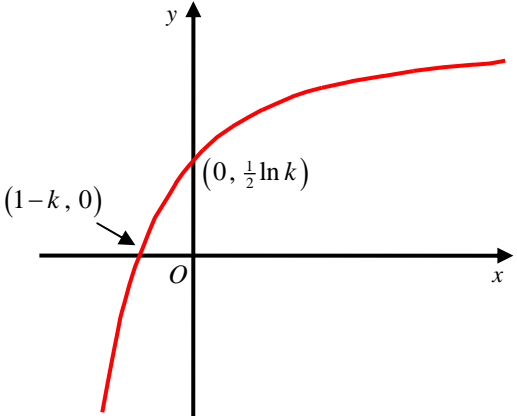
Question Number	Scheme	Marks
3.	$P = 80e^{\frac{t}{5}}$	
(a)	$t = 0 \Rightarrow P = 80e^{\frac{0}{5}} = 80(1) = \underline{80}$	B1 [1]
(b)	$P = 1000 \Rightarrow 1000 = 80e^{\frac{t}{5}} \Rightarrow \frac{1000}{80} = e^{\frac{t}{5}}$ $\therefore t = 5 \ln\left(\frac{1000}{80}\right)$ $t = 12.6286\dots$	M1
	<div style="border: 1px solid black; padding: 2px; display: inline-block;">awrt 12.6 or 13 years</div>	A1 [2]
(c)	$\frac{dP}{dt} = 16e^{\frac{t}{5}}$	M1 $ke^{\frac{1}{5}t}$ and $k \neq 80$ . A1 $16e^{\frac{1}{5}t}$ [2]
(d)	$50 = 16e^{\frac{t}{5}}$ $\therefore t = 5 \ln\left(\frac{50}{16}\right) \quad \{= 5.6797\dots\}$ $P = 80e^{\frac{1}{5}\left(5 \ln\left(\frac{50}{16}\right)\right)}$ or $P = 80e^{\frac{1}{5}(5.6797\dots)}$	M1 Using $50 = \frac{dP}{dt}$ and an attempt to solve to find the value of $t$ or $\frac{t}{5}$ .
	$P = \frac{80(50)}{16} = \underline{250}$	dM1 Substitutes their value of $t$ back into the equation for $P$ . A1 $\underline{250}$ or awrt 250 [3]
		<b>8 marks</b>

Note  $t = 12$   
 or  $t = \text{awrt } 12.6 \Rightarrow t = 12$  will score A0.

Question Number	Scheme	Marks
4. (i) (a)	$y = x^2 \cos 3x$ <p>Apply product rule: <math>\left\{ \begin{array}{l} u = x^2 \quad v = \cos 3x \\ \frac{du}{dx} = 2x \quad \frac{dv}{dx} = -3 \sin 3x \end{array} \right\}</math></p> $\frac{dy}{dx} = 2x \cos 3x - 3x^2 \sin 3x$	<p>Applies <math>vu' + uv'</math> correctly for their <math>u, u', v, v'</math> AND gives an expression of the form <math>\alpha x \cos 3x \pm \beta x^2 \sin 3x</math></p> <p>Any one term correct</p> <p>Both terms correct and no further simplification to terms in <math>\cos \alpha x^2</math> or <math>\sin \beta x^3</math>.</p> <p>M1 A1 A1</p> <p>[3]</p>
(b)	$y = \frac{\ln(x^2 + 1)}{x^2 + 1}$ <p><math>u = \ln(x^2 + 1) \Rightarrow \frac{du}{dx} = \frac{2x}{x^2 + 1}</math></p> <p>Apply quotient rule: <math>\left\{ \begin{array}{l} u = \ln(x^2 + 1) \quad v = x^2 + 1 \\ \frac{du}{dx} = \frac{2x}{x^2 + 1} \quad \frac{dv}{dx} = 2x \end{array} \right\}</math></p> $\frac{dy}{dx} = \frac{\left(\frac{2x}{x^2 + 1}\right)(x^2 + 1) - 2x \ln(x^2 + 1)}{(x^2 + 1)^2}$ $\left\{ \frac{dy}{dx} = \frac{2x - 2x \ln(x^2 + 1)}{(x^2 + 1)^2} \right\}$	<p><math>\ln(x^2 + 1) \rightarrow \frac{\text{something}}{x^2 + 1}</math></p> <p><math>\ln(x^2 + 1) \rightarrow \frac{2x}{x^2 + 1}</math></p> <p>M1 A1</p> <p>Applying <math>\frac{vu' - uv'}{v^2}</math></p> <p>Correct differentiation with correct bracketing but allow recovery.</p> <p>M1 A1</p> <p>{Ignore subsequent working.}</p> <p>[4]</p>

Question Number	Scheme	Marks
4. (ii)	<p><math>y = \sqrt{4x+1}, x &gt; -\frac{1}{4}</math></p> <p>At <math>P, y = \sqrt{4(2)+1} = \underline{\underline{\sqrt{9}}} = \underline{\underline{3}}</math></p> <p><math>\frac{dy}{dx} = \frac{1}{2}(4x+1)^{-\frac{1}{2}}(4)</math></p> <p><math>\frac{dy}{dx} = \frac{2}{(4x+1)^{\frac{1}{2}}}</math></p> <p>At <math>P, \frac{dy}{dx} = \frac{2}{(4(2)+1)^{\frac{1}{2}}}</math></p> <p>Hence <math>m(\mathbf{T}) = \frac{2}{3}</math></p> <p>Either <math>\mathbf{T}: y - 3 = \frac{2}{3}(x - 2);</math>  or <math>y = \frac{2}{3}x + c</math> and  <math>3 = \frac{2}{3}(2) + c \Rightarrow c = 3 - \frac{4}{3} = \frac{5}{3};</math></p> <p>Either <math>\mathbf{T}: 3y - 9 = 2(x - 2);</math>  <math>\mathbf{T}: 3y - 9 = 2x - 4</math>  <math>\mathbf{T}: \underline{\underline{2x - 3y + 5 = 0}}</math>  or <math>\mathbf{T}: y = \frac{2}{3}x + \frac{5}{3}</math>  <math>\mathbf{T}: 3y = 2x + 5</math>  <math>\mathbf{T}: \underline{\underline{2x - 3y + 5 = 0}}</math></p>	<p>At <math>P, y = \underline{\underline{\sqrt{9}}}</math> or <math>\underline{\underline{3}}</math></p> <p><math>\pm k(4x+1)^{-\frac{1}{2}}</math></p> <p><math>2(4x+1)^{-\frac{1}{2}}</math></p> <p>Substituting <math>x = 2</math> into an equation involving <math>\frac{dy}{dx};</math></p> <p><math>y - y_1 = m(x - 2)</math>  or <math>y - y_1 = m(x - \text{their stated } x)</math> with ‘their TANGENT gradient’ and their <math>y_1</math>;  or uses <math>y = mx + c</math> with ‘their TANGENT gradient’, their <math>x</math> and their <math>y_1</math>.</p> <p><math>\underline{\underline{2x - 3y + 5 = 0}}</math></p> <p><b>13 marks</b></p>

Tangent must be stated in the form  $ax + by + c = 0$ , where  $a, b$  and  $c$  are integers.

Question Number	Scheme	Marks
5. (a)		<p>Curve retains shape when <math>x &gt; \frac{1}{2} \ln k</math> B1</p> <p>Curve reflects through the <math>x</math>-axis when <math>x &lt; \frac{1}{2} \ln k</math> B1</p> <p><math>(0, k-1)</math> and <math>(\frac{1}{2} \ln k, 0)</math> marked in the correct positions. B1</p> <p>[3]</p>
(b)		<p>Correct shape of curve. The curve should be contained in quadrants 1, 2 and 3 (Ignore asymptote) B1</p> <p><math>(1-k, 0)</math> and <math>(0, \frac{1}{2} \ln k)</math> B1</p> <p>[2]</p>
(c)	<p>Range of <math>f</math>: <math>f(x) &gt; -k</math> or <math>y &gt; -k</math> or <math>(-k, \infty)</math></p>	<p>Either <math>f(x) &gt; -k</math> or <math>y &gt; -k</math> or <math>(-k, \infty)</math> or <math>f &gt; -k</math> or Range <math>&gt; -k</math>. B1</p> <p>[1]</p>
(d)	<p><math>y = e^{2x} - k \Rightarrow y + k = e^{2x}</math>  <math>\Rightarrow \ln(y + k) = 2x</math>  <math>\Rightarrow \frac{1}{2} \ln(y + k) = x</math></p> <p>Hence <math>f^{-1}(x) = \frac{1}{2} \ln(x + k)</math></p>	<p>Attempt to make <math>x</math> (or swapped <math>y</math>) the subject M1</p> <p>Makes <math>e^{2x}</math> the subject and takes <math>\ln</math> of both sides M1</p> <p><math>\frac{1}{2} \ln(x + k)</math> A1 cao [3]</p>
(e)	<p><math>f^{-1}(x)</math>: Domain: <math>x &gt; -k</math> or <math>(-k, \infty)</math></p>	<p>Either <math>x &gt; -k</math> or <math>(-k, \infty)</math> or Domain <math>&gt; -k</math> or <math>x</math> "ft one sided inequality" their part (c) RANGE answer (see appendix) B1 <math>\sqrt{\quad}</math> [1]</p>
		<b>10 marks</b>

Question Number	Scheme	Marks
6. (a)	$A = B \Rightarrow \cos(A + A) = \cos 2A = \underline{\cos A \cos A - \sin A \sin A}$ <p>Applies <math>A = B</math> to <math>\cos(A + B)</math> to give the <u>underlined</u> equation or <math>\cos 2A = \underline{\cos^2 A - \sin^2 A}</math></p> $\cos 2A = \cos^2 A - \sin^2 A \quad \text{and} \quad \cos^2 A + \sin^2 A = 1 \text{ gives}$ $\underline{\cos 2A} = 1 - \sin^2 A - \sin^2 A = \underline{1 - 2\sin^2 A} \quad (\text{as required})$	M1 A1 <b>AG</b> [2]
6. (b)	$C_1 = C_2 \Rightarrow 3\sin 2x = 4\sin^2 x - 2\cos 2x$ <p>Eliminating <math>y</math> correctly.</p> $3\sin 2x = 4\left(\frac{1 - \cos 2x}{2}\right) - 2\cos 2x$ <p>Using result in part (a) to substitute for <math>\sin^2 x</math> as <math>\frac{\pm 1 \pm \cos 2x}{2}</math> or <math>k \sin^2 x</math> as <math>k\left(\frac{\pm 1 \pm \cos 2x}{2}\right)</math> to produce an equation in only double angles.</p> $3\sin 2x = 2(1 - \cos 2x) - 2\cos 2x$ $3\sin 2x = 2 - 2\cos 2x - 2\cos 2x$ $3\sin 2x + 4\cos 2x = 2$ <p>Rearranges to give correct result</p>	M1 M1 A1 <b>AG</b> [3]
6. (c)	$3\sin 2x + 4\cos 2x = R\cos(2x - \alpha)$ $3\sin 2x + 4\cos 2x = R\cos 2x \cos \alpha + R\sin 2x \sin \alpha$ <p>Equate <math>\sin 2x</math>: <math>3 = R\sin \alpha</math> Equate <math>\cos 2x</math>: <math>4 = R\cos \alpha</math></p> $R = \sqrt{3^2 + 4^2}; = \sqrt{25} = 5$ <p><math>R = 5</math></p> $\tan \alpha = \frac{3}{4} \Rightarrow \alpha = 36.86989765\dots^\circ$ <p><math>\tan \alpha = \pm \frac{3}{4}</math> or <math>\tan \alpha = \pm \frac{4}{3}</math> or <math>\sin \alpha = \pm \frac{3}{\text{their } R}</math> or <math>\cos \alpha = \pm \frac{4}{\text{their } R}</math> awrt 36.87</p> <p>Hence, <math>3\sin 2x + 4\cos 2x = 5\cos(2x - 36.87)</math></p>	B1 M1 A1 [3]

Question Number	Scheme	Marks
6. (d)	$3\sin 2x + 4\cos 2x = 2$ $5\cos(2x - 36.87) = 2$ $\cos(2x - 36.87) = \frac{2}{5}$ $(2x - 36.87) = 66.42182\dots^\circ$ $(2x - 36.87) = 360 - 66.42182\dots^\circ$ <p>Hence, <math>x = 51.64591\dots^\circ, 165.22409\dots^\circ</math></p>	$\cos(2x \pm \text{their } \alpha) = \frac{2}{\text{their } R}$ M1 awrt 66 A1 One of either awrt 51.6 or awrt 51.7 or awrt 165.2 or awrt 165.3 A1 Both of awrt 51.6 or awrt 165.2 A1 <b>[4]</b> <b>12 marks</b>

If there are any EXTRA solutions inside the range  $0 \leq x < 180^\circ$  then withhold the final accuracy mark.  
Also ignore EXTRA solutions outside the range  $0 \leq x < 180^\circ$ .



Question Number	Scheme	Marks
<p><b>7.</b></p> <p>(a)</p>	$f(x) = 1 - \frac{2}{(x+4)} + \frac{x-8}{(x-2)(x+4)}$ <p><math>x \in \mathbb{R}, x \neq -4, x \neq 2.</math></p> $f(x) = \frac{(x-2)(x+4) - 2(x-2) + x-8}{(x-2)(x+4)}$ $= \frac{x^2 + 2x - 8 - 2x + 4 + x - 8}{(x-2)(x+4)}$ $= \frac{x^2 + x - 12}{[(x+4)(x-2)]}$ $= \frac{(x+4)(x-3)}{[(x+4)(x-2)]}$ $= \frac{(x-3)}{(x-2)}$	<p>An attempt to combine to one fraction <b>M1</b></p> <p>Correct result of combining all three fractions <b>A1</b></p> <p>Simplifies to give the correct numerator. Ignore omission of denominator <b>A1</b></p> <p>An attempt to factorise the numerator. See rules for factorising quadratics. <b>dM1</b></p> <p>Correct result <b>A1 cso AG</b></p> <p><b>[5]</b></p>
<p>(b)</p>	$g(x) = \frac{e^x - 3}{e^x - 2} \quad x \in \mathbb{R}, x \neq \ln 2.$ <p>Apply quotient rule: <math>\left\{ \begin{array}{l} u = e^x - 3 \quad v = e^x - 2 \\ \frac{du}{dx} = e^x \quad \frac{dv}{dx} = e^x \end{array} \right\}</math></p> $g'(x) = \frac{e^x(e^x - 2) - e^x(e^x - 3)}{(e^x - 2)^2}$ $= \frac{e^{2x} - 2e^x - e^{2x} + 3e^x}{(e^x - 2)^2}$ $= \frac{e^x}{(e^x - 2)^2}$	<p>Applying <math>\frac{vu' - uv'}{v^2}</math> <b>M1</b></p> <p>Correct differentiation <b>A1</b></p> <p>Correct result <b>A1 AG cso</b></p> <p><b>[3]</b></p>

Question Number	Scheme	Marks
7. (c)	$g'(x) = 1 \Rightarrow \frac{e^x}{(e^x - 2)^2} = 1$ $e^x = (e^x - 2)^2$ $e^x = e^{2x} - 2e^x - 2e^x + 4$ $\underline{e^{2x} - 5e^x + 4 = 0}$ $(e^x - 4)(e^x - 1) = 0$ $e^x = 4 \text{ or } e^x = 1$ $x = \ln 4 \text{ or } x = 0$	<p>Puts their differentiated numerator equal to their denominator. M1</p> <p><math>\underline{e^{2x} - 5e^x + 4}</math> A1</p> <p>Attempt to factorise or solve quadratic in <math>e^x</math> M1</p> <p>both <math>x = 0, \ln 4</math> A1</p> <p style="text-align: right;"><b>[4]</b></p> <hr/> <p><b>12 marks</b></p>

Question Number	Scheme	Marks
8. (a)	$\sin 2x = \underline{2 \sin x \cos x}$	$\underline{2 \sin x \cos x}$ B1 aef [1]
8. (b)	$\operatorname{cosec} x - 8 \cos x = 0, \quad 0 < x < \pi$  $\frac{1}{\sin x} - 8 \cos x = 0$  $\frac{1}{\sin x} = 8 \cos x$  $1 = 8 \sin x \cos x$  $1 = 4(2 \sin x \cos x)$  $1 = 4 \sin 2x$  $\underline{\sin 2x = \frac{1}{4}}$  Radians $2x = \{0.25268\dots, 2.88891\dots\}$ Degrees $2x = \{14.4775\dots, 165.5225\dots\}$  Radians $x = \{0.12634\dots, 1.44445\dots\}$ Degrees $x = \{7.23875\dots, 82.76124\dots\}$	Using $\operatorname{cosec} x = \frac{1}{\sin x}$ M1           $\sin 2x = k$ , where $-1 < k < 1$ M1 $\underline{\sin 2x = \frac{1}{4}}$ A1     Either arwt 7.24 or 82.76 or 0.13 or 1.44 or 1.45 or awrt $0.04\pi$ or awrt $0.46\pi$ . A1 Both <u>0.13</u> and <u>1.44</u> A1 cao [5]
<b>6 marks</b>		

Solutions for the final two A marks must be given in  $x$  only.

If there are any EXTRA solutions inside the range  $0 < x < \pi$  then withhold the final accuracy mark. Also ignore EXTRA solutions outside the range  $0 < x < \pi$ .