

Paper Reference(s)

6665

Edexcel GCE

Core Mathematics C3

Advanced Level

Thursday 15 January 2009 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination

Mathematical Formulae (Green)

Items included with question papers

Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, other name and signature.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

Full marks may be obtained for answers to ALL questions.

The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).

There are 8 questions on this paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. a) Find the value of $\frac{dy}{dx}$ at the point where $x = 2$ on the curve with equation

$$y = x^2 \sqrt{5x - 1}.$$

(6)

- (b) Differentiate $\frac{\sin 2x}{x^2}$ with respect to x .

(4)

2.

$$f(x) = \frac{2x+2}{x^2-2x-3} - \frac{x+1}{x-3}$$

- (a) Express $f(x)$ as a single fraction in its simplest form.

(4)

- (b) Hence show that $f'(x) = \frac{2}{(x-3)^2}$

(3)

3.

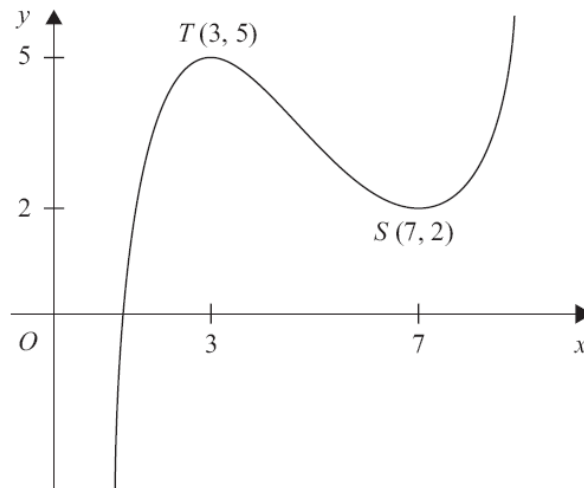


Figure 1

Figure 1 shows the graph of $y = f(x)$, $1 < x < 9$.
The points $T(3, 5)$ and $S(7, 2)$ are turning points on the graph.
Sketch, on separate diagrams, the graphs of

- (a) $y = 2f(x) - 4$,

(3)

- (b) $y = |f(x)|$.

(3)

Indicate on each diagram the coordinates of any turning points on your sketch.

4. Find the equation of the tangent to the curve $x = \cos(2y + \pi)$ at $\left(0, \frac{\pi}{4}\right)$

Give your answer in the form $y = ax + b$, where a and b are constants to be found.

(6)

5. The functions f and g are defined by

$$f : x \mapsto 3x + \ln x, x > 0, x \in \mathfrak{R}$$

$$g : x \mapsto e^{x^2}, x \in \mathfrak{R}$$

(a) Write down the range of g .

(1)

(b) Show that the composite function fg is defined by $fg : x \mapsto x^2 + 3e^{x^2}$

(2)

(c) Write down the range of fg .

(1)

(d) Solve the equation $\frac{d[fg(x)]}{dx} = x(xe^{x^2} + 2)$.

(6)

6. (a) (i) By writing $3\theta = (2\theta + \theta)$, show that

$$\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta.$$

(4)

(ii) Hence, or otherwise, solve

$$8 \sin^3 \theta - 6 \sin \theta + 1 = 0.$$

Give your answers in terms of π .

(5)

(b) Using $\sin(\theta - \alpha) = \sin \theta \cos \alpha - \cos \theta \sin \alpha$, or otherwise, show that

$$\sin 15^\circ = \frac{1}{4} (\sqrt{6} - \sqrt{2}).$$

(4)

7. $f(x) = 3xe^x - 1$

The curve with equation $y = f(x)$ has a turning point P .

(a) Find the exact coordinates of P .

(5)

The equation $f(x) = 0$ has a root between $x = 0.25$ and $x = 0.3$

(b) Use the iterative formula $x_{n+1} = \frac{1}{3}e^{-x_n}$

with $x_0 = 0.25$ to find, to 4 decimal places, the values of x_1 , x_2 and x_3 .

(3)

(c) By choosing a suitable interval, show that a root of $f(x) = 0$ is $x = 0.2576$ correct to 4 decimal places.

(3)

8. (a) Express $3 \cos \theta + 4 \sin \theta$ in the form $R \cos(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90^\circ$. (4)

- (b) Hence find the maximum value of $3 \cos \theta + 4 \sin \theta$ and the smallest positive value of θ for which this maximum occurs. (3)

The temperature, $f(t)$, of a warehouse is modelled using the equation

$$f(t) = 10 + 3 \cos(15t)^\circ + 4 \sin(15t)^\circ,$$

where t is the time in hours from midday and $0 \leq t < 24$.

- (c) Calculate the minimum temperature of the warehouse as given by this model. (2)

- (d) Find the value of t when this minimum temperature occurs. (3)

TOTAL FOR PAPER: 75 MARKS

END