

Mark Scheme Summer 2008

AEA

AEA Mathematics (9801)

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①. $a=200, d=-\frac{5}{2}$ $u_n = 0 \Rightarrow 200 - \frac{5}{2}(n-1) = 0$

$\Rightarrow n = 81$

[ALT $S_n = \frac{n}{2}(400 - \frac{5}{2}(n-1))$



Max at 80.5

Identify a, d and set $u_n = 0$

M1
A1

S_n and attempt max

M1
A1

Maximum sum when $n=80$ or 81

$S_{80} = 40 [400 - \frac{5}{2} \times 79]$

$= 20 [800 - 395]$

$= \underline{\underline{8100}}$

Use of S_n with $n=80$ or $n=81$

M1 A1

A1

⑤

②. (a) $\frac{dy}{dx} = 2 \Rightarrow 2(x+1)(x+2) = xy$

$\Rightarrow 2(x^2 + 3x + 2) = x(2x+5)$

$y = 2x+5 \Rightarrow \underline{\underline{x = -4, y = -3}}$ [or P is (-4, -3)]

(b) $\int \frac{1}{y} dy = \int \frac{x}{(x+1)(x+2)} dx$

$= \int (\frac{2}{x+2} - \frac{1}{x+1}) dx$

$\Rightarrow \ln|y| = 2\ln|x+2| - \ln|x+1| + C$

$\ln y = \ln \left[\frac{A(x+2)^2}{(x+1)} \right] + C$ or $\ln \left[\frac{(x+2)^2}{(x+1)} \right] + C$

$y = \frac{A(x+2)^2}{(x+1)}$

Using P(-4, -3) $\Rightarrow -3 = \frac{A(-2)^2}{(-3)}$

$\underline{\underline{y = \frac{9(x+2)^2}{4(x+1)}}}$

Sub $\frac{dy}{dx} = 2$

M1

sub y for $2x+5$ and attempt to solve

M1

A1

A1

(4)

Separation attempt

M1

Attempt partial fractions

M1

Some correct \ln integral of x function

M1 A1

Use of log rules

M1

$\rightarrow \ln[g(x)]$ (condone $A=1$ or $C=0$)

Getting out of loss (must have 'A' or equiv)

M1

Use P to form eqn in A or C

M1

A1

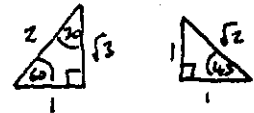
(8)

Max S1 only for 11 or 12/12

⑫

S1 or S2 For a fully correct (or nearly so) and neat or succinct solution to Qn 2 - Qn 7. Count best 3 questions.

T1 For a good attempt at the whole paper (all questions)

③ (a) $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$; $t = \tan 15$ 

$\tan 30 = \frac{1}{\sqrt{3}} \Rightarrow \frac{1}{\sqrt{3}} = \frac{2t}{1-t^2}$

$t^2 + 2\sqrt{3}t - 1 = 0 \Rightarrow t = \frac{-2\sqrt{3} \pm \sqrt{12+4}}{2}$

$t = \tan 15 = \underline{2-\sqrt{3}}$ (*)

Use of known t_{15} ... BI
 Use ~~tan~~ → MI
 equation in t
 Attempt to solve → t: MI
 [S for considerations ±] AI csp (4)

(b) $\left(\frac{\sin\theta}{2} + \frac{\sqrt{3}\cos\theta}{2}\right)\left(\frac{\sin\theta}{2} - \frac{\sqrt{3}\cos\theta}{2}\right) = \cos^2\theta(1-\sqrt{3})$

$\frac{\sin^2\theta}{4} - \frac{3}{4}\cos^2\theta = \cos^2\theta - \sqrt{3}\cos^2\theta$

$\frac{\sin^2\theta}{4} = \frac{\cos^2\theta}{4}(7-4\sqrt{3})$

$\cos^2\theta = \frac{1}{4(2-\sqrt{3})}$ or $\frac{2+\sqrt{3}}{4}$ or $\tan^2\theta = 7-4\sqrt{3}$ or $\cos 2\theta = \frac{2\sqrt{3}-3}{4-2\sqrt{3}}$

$\tan^2\theta = (2-\sqrt{3})^2$

$\tan\theta = \pm(2-\sqrt{3})$ or $\cos 2\theta = \frac{\sqrt{3}}{2}$

$\tan\theta = 2-\sqrt{3} \Rightarrow \theta = \underline{15, 195}$; $\tan\theta = -(2-\sqrt{3}) \Rightarrow \theta = \underline{165, 345}$

Use of $\sin(A \pm B)$ MI
 Equation in s^2 and c^2 MI
 or c^2 and $\cos 2\theta$
 Attempt $\cos^2\theta$, $\tan^2\theta$ MI
 or $\cos 2\theta$ or $\sin^2\theta$
 AI
 $(2-\sqrt{3})^2 = 7-4\sqrt{3}$ MI
 AI
 AI; AI (8) (12)

④ (a) $\frac{dy}{dx} = -\sin x \ln(\sec x) + \cos x \cdot \tan x$


$y' = 0 \Rightarrow 0 = \sin x (1 - \ln(\sec x))$

$\sin x = 0 \Rightarrow x = 0 \therefore \text{Min at origin}$

$\ln \sec x = 1 \Rightarrow \sec x = e ; \therefore \theta \in (\arccos \frac{1}{e}, \frac{1}{e})$

(2) [Rectangle - S]

Use of product rule MI AI
 Take out $\sin x$ factor MI
 [Smokes]
 AI; AI (5)
 For strategies MI
 Attempt parts MI AI

(b) 

$I = \int \cos x \ln(\sec x) dx = \sin x \ln \sec x - \int \sin x \tan x dx$

$I = \sin x \ln \sec x - \int \frac{\sin^2 x}{\cos x} dx = \sin x \ln \sec x - \int (\sec x - \cos x) dx$

$I = \sin x \ln \sec x - \ln|\sec x + \tan x| + \frac{\sin x}{\cos x}$

$S = [I]_0^{\arccos \frac{1}{e}}$

$S = \frac{\sqrt{e^2-1}}{e} - \ln[e + \sqrt{e^2-1}] + \frac{\sqrt{e^2-1}}{e}$

$\text{Area} = 2\left[\frac{1}{e} \arccos \frac{1}{e} - S\right] = \underline{\underline{\frac{2}{e} \arccos \frac{1}{e} + 2\ln(e + \sqrt{e^2-1}) - \frac{4\sqrt{e^2-1}}{e}}}$ (*)

Put $\sin x \tan x$ into integrable form MI
 correct integration AI AI
 Attempt correct limits and $\sin x$ and $\cos x$ in terms of $e, \sqrt{e^2-1}$ etc. MI
 Must have complete integral
 AI csp (8)
 (13)

5 (i) $(\log_3 p)^2 = \log_3(p^2) \Rightarrow (\log_3 p)^2 = 2 \log_3 p$
 $\Rightarrow \log_3 p (\log_3 p - 2) = 0 \Rightarrow \log_3 p = 0 \therefore p = 1$
 or $\log_3 p = 2 \therefore p = 9$
 $\log_3(p+q) = \log_3 p + \log_3 q \Rightarrow \log_3(p+q) = \log_3(pq)$
 $\Rightarrow p+q = pq$ or $q+q = 9q$
 $\therefore q = \frac{p}{p-1} \quad (p \neq 1)$
 $p = 9 \Rightarrow q = \frac{9}{8}$

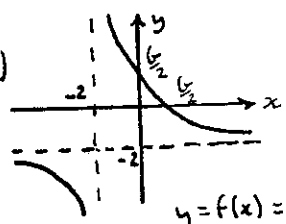
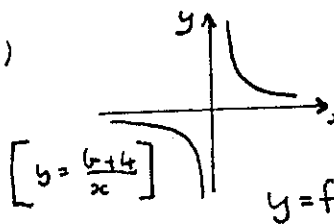
Use $n \log x$ rule
 Use of $\log x + \log y$ rule

M1
 A1
 A1
 M1
 A1

(ii) $\log_3 \left[\frac{3x^3 - 23x^2 + 40x}{(3x-8)^2} \right] = 1$
 $\frac{3x^3 - 23x^2 + 40x}{(3x-8)^2} = 3$
 $\frac{x(3x-8)(x-5)}{(3x-8)^2} = 3$
 $x^2 - 5x = 9x - 24 \Rightarrow x^2 - 14x + 24 = 0$
 $(x-2)(x-12) = 0 \Rightarrow x = 2 \text{ or } 12$
 $(x = 9/8 \text{ listed here loses final A1})$
 $\log_3 9 = 2 \text{ o.e.}$

Making q the subject
 [S for $p \neq 1$]
 Seen anywhere
 Use of log rules to form a single log out of logs
 For reducing cubic equation to quadratic
 [$x \neq 8/3$ for S mob]
 3TQ (accused 3TQ)
 (ignore $x=2$ and $9/8$)
 [Smobs for connection]

M1
 A1 (7)
 B1
 M1
 M1
 M1
 A1
 M1 ; A1 (7)
 (14)

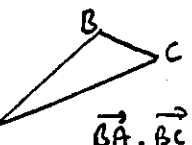
6 (a) $yx + 2y = ax + b \Rightarrow x = \frac{b-2y}{y-a} \therefore f^{-1}(x) = \frac{b-2x}{x-a}$
 (b) $f(f(x)) = x \Rightarrow f^{-1}(b) = f(x) ; \therefore a = -2$
 (c) (i) 
 (ii) 
 $y = f(x) = \frac{b-2x}{x+2}$
 $y = f(x-2) + 2$
 (d) Normal at P' on $y = f(x-2) + 2$ is: $y = 4(x-2) - 39 + 2$
 $y = 4x - 45$
 curve is symmetric about $y = \frac{1}{2}x$, normal at Q' will be $y = 4x + 45$
 [symmetry is $x \rightarrow -x$ and $y \rightarrow -y$]
 Reversing process
 Normal at Q on $y = f(x)$ is: $y = 4(x+2) + 45 - 2$
 $\therefore y = 4x + 51 \text{ or } k = 51$

Make x the subject
 $f^{-1} = f$
 shape
 $x = -2; y = -2$
 $(\frac{1}{2}, 0), (0, \frac{1}{2})$
 $\rightarrow +2$
 $\uparrow +2$
 both branches
 Use transformations on normal
 Use symmetry on Q'
 Use $f(x+2) - 2$

M1, A1 (2)
 M1; A1 (2)
 B1 (no overlap)
 B1
 B1 (3)
 M1
 M1
 A1 (3)
 M1
 A1
 M1
 A1 (5)

[NB: P' is (12, 3)
 P is (10, 1); $b = 32$; Q = (-14, -5)]
 $y - 5 = 4(x - 14)$
 $\rightarrow k = 51$

M1
 A1

(7) (a)  $\vec{BA} = \begin{pmatrix} -4 \\ 1 \\ -8 \end{pmatrix}$ $\vec{BC} = \begin{pmatrix} 4 \\ 2 \\ -4 \end{pmatrix}$
 $\vec{BA} \cdot \vec{BC} = -16 + 2 + 32 = 18$
 $|\vec{BA}| = \sqrt{4^2 + 1^2 + 8^2} = 9$, $|\vec{BC}| = \sqrt{4^2 + 2^2 + 4^2} = 6$
 $\cos \theta = \frac{18}{9 \times 6} = \frac{1}{3}$

Attempt \vec{BA} and \vec{BC}

M1

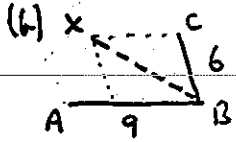
Attempt $\vec{BA} \cdot \vec{BC}$

M1

Attempt $|\vec{BA}|$ or $|\vec{BC}|$

M1

AI (4)



Using rhombus idea, $\vec{BX} = \vec{BC} + \frac{2}{3}\vec{BA}$ o.e.

$$= \lambda \begin{pmatrix} 4 \\ 8 \\ -28 \end{pmatrix} \text{ or } \mu \begin{pmatrix} 1 \\ 2 \\ -7 \end{pmatrix}$$

eg $3\vec{BC} + 2\vec{BA}$
Any correct ratio

M1, AI

AI

AI c/o.

(4)

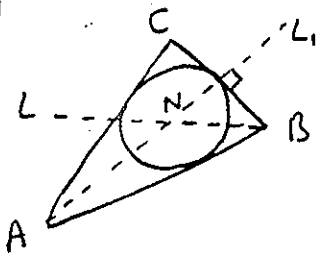
Through B \therefore
$$\underline{\underline{r = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ -7 \end{pmatrix} \quad (*)}}$$

(c) $\vec{AC} = \begin{pmatrix} 8 \\ 1 \\ 4 \end{pmatrix}$ $|\vec{AC}| = \sqrt{8^2 + 1^2 + 4^2} = 9 = |\vec{BA}|$

Attempt \vec{AC} and $|\vec{AC}|$

M1 AI c/o (2)

(must say = $|\vec{BA}|$ for AI)



(d) \therefore ABC is isos L_1 has direction $\frac{1}{2}(\vec{AB} + \vec{AC}) = \begin{pmatrix} 6 \\ 0 \\ 6 \end{pmatrix}$

BI

$\therefore L_1$ has equation $(r =) \begin{pmatrix} -3 \\ 1 \\ -9 \end{pmatrix} + u \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

Find equation of L_1

M1

Centre of S is intersection of L_1 and L

Strategy

M1

Solving: $\left. \begin{matrix} 1+t = -3+u \\ 2t = 1 \end{matrix} \right\} \Rightarrow t = \frac{1}{2}, u = \frac{9}{2}$

Attempt to solve $t = \frac{1}{2}, u = \frac{9}{2}$

M1

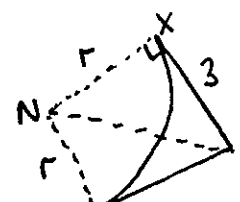
AI

$\left[-1-7t = -9+u \quad \text{Check: LHS} = -\frac{9}{2}, \text{RHS} = -\frac{9}{2} \right]$

\therefore Centre has position vector $\underline{\underline{\vec{ON} = \begin{pmatrix} 3/2 \\ 1 \\ -9/2 \end{pmatrix}}}$

(-100)

A2/1/0 (7)

(e)  let X = mid-point of BC $\therefore BX = 3$ (\therefore isos)
 $\vec{BN} = \begin{pmatrix} 1/2 \\ 1 \\ -7/2 \end{pmatrix}$ $\therefore |\vec{BN}| = \frac{1}{2}\sqrt{54}$

$BX = 3$

BI

Attempt $|\vec{BN}| = |\vec{BN}|$

M1

AI

$r^2 = |\vec{BN}|^2 - 3^2 \therefore r^2 = \frac{54}{4} - 9 \therefore r = \frac{\sqrt{18}}{2} \text{ or } \frac{3\sqrt{2}}{2}$

Full method for r

M1

AI (5)

(22)

Final
 CB Afford
 1/7/08

3(a) $\tan 15 = \tan(60-45) = \frac{\tan 60 - \tan 45}{1 + \tan 60 \tan 45} = \frac{\sqrt{3} - 1}{1 + \sqrt{3}}$
 $= \frac{(\sqrt{3}-1)^2}{3-1}, = \frac{4-2\sqrt{3}}{2} = \underline{\underline{2-\sqrt{3}}}$

or $\tan(45-30)$

NB no need for \pm consideration this way

BI MI

MI, AI (4)

3(b) Use of $\cos A - \cos B = \dots = 2 \sin\left(\frac{20+120}{2}\right) \sin\left(\frac{20-120}{2}\right)$

$\rightarrow -\frac{1}{2} [\cos 20 - \cos 120] = (1-\sqrt{3}) \cos^2 \theta$

$\rightarrow \cos 20 + \frac{1}{2} = 2(\sqrt{3}-1) \cos^2 \theta$

May then go for $\cos^2 \theta = \dots$ or $\cos 2\theta = \dots$ as per scheme.

See snippets

3A

MI

MI

6(d) For a valid method $\rightarrow P = (10, 1)$ or $P' = (12, 3)$

For obtaining $Q = (-14, -5)$

Equation of normal: $y - 5 = 4(x - 14)$

$y = 4x + 51$ or $k = 51$

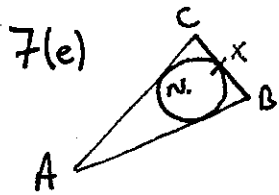
Requires $b = 32$ on the way!

MI AI

MI

MI

AI (5)



Midpoint of BC, $\vec{OX} = \begin{pmatrix} 3 \\ -3 \end{pmatrix}$

$\vec{NX} = \pm \begin{pmatrix} 3/2 \\ 0 \\ 3/2 \end{pmatrix}$

$r = |\vec{NX}| = \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right)^2} = \underline{\underline{\frac{3}{2}\sqrt{2}}}$ o.e.

Must be in (c)

BI

MI AI

MI AI (5)

7(d) $\vec{ON} = \begin{pmatrix} 1+t \\ 2t \\ -1-7t \end{pmatrix}$

$\vec{AN} = \begin{pmatrix} 4+t \\ -1+2t \\ 8-7t \end{pmatrix}$

$\vec{AN} \cdot \vec{BC} = 0 \Rightarrow 16+4t - 2+4t - 32+28t = 0$
 $36t = 18$
 $t = \frac{1}{2}$

General $\vec{ON} =$

BI

Attempt \vec{AN} in t

MI

$\vec{AN} \cdot \vec{BC} = 0$

MI

Solve for t

MI

Then as before.

2(b) $\ln|y| = \frac{1}{2} \int \frac{2x+3}{x^2+3x+2} dx - \frac{3}{2} \int \frac{1}{(x+\frac{1}{2})^2 + (\frac{1}{2})^2} dx$
 $= \frac{1}{2} \ln|x^2+3x+2| - 3 \ln \left| \frac{x+\frac{3}{2}-\frac{1}{2}}{x+\frac{3}{2}+\frac{1}{2}} \right| + c$
 $= \dots$

sep
 Equiv to PIF

MI

MI

Correct ln integration

MI

AI

etc.

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