Mark Scheme (Results)

June 2013

GCE Further Pure Mathematics FP1 (6667/01)
Original Paper

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications come from Pearson, the world's leading learning company. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information, please visit our website at www.edexcel.com.

Our website subject pages hold useful resources, support material and live feeds from our subject advisors giving you access to a portal of information. If you have any subject specific questions about this specification that require the help of a subject specialist, you may find our Ask The Expert email service helpful.

www.edexcel.com/contactus

Pearson: helping people progress, everywhere

Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

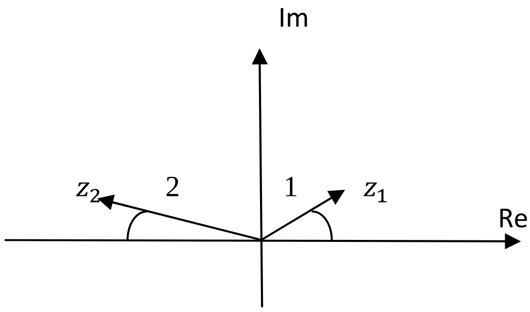
Summer 2012
Publications Code N/A

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

Question Number	Scheme	Marks
1(a)	det $\mathbf{M} = a(2-a)-1$	M1A1
		(2)
1(b)	det $\mathbf{M} = 0$	M1
	$a^2 - 2a + 1 = 0$	
	$(a-1)^2 = 0$	M1
	$a = 1$	A1
		(3)
		[5]
	Notes	
(a)	M for " $ad - bc$ "	
(b)	First M for their det $\mathbf{M} = 0$	
	Second M for attempt to solve their 3 term quadratic	
	Method mark for solving 3 term quadratic:	
	1. Factorisation	
	$(x^2 + bx + c) = (x + p)(x + q)$, where $\lvert pq \rvert = \lvert c \rvert$, leading to x =	
	$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $\lvert pq \rvert = \lvert c \rvert$ and $\lvert mn \rvert = \lvert a \rvert$,leading to x =	
	2. Formula	
	Attempt to use correct formula (with values for a, b and c).	
	3. Completing the square	
	Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c, \quad q \neq 0$, leading to $x = \ldots$	

Question Number	Scheme	Marks
2	$z=-2i-1$ is also a root	B1
	$\left(z-\left(2i-1\right)\right)\left(z-\left(-2i-1\right)\right)=z^2+2z+5$	M1A1
	$\left(z+3\right)\left(z^2+2z+5\right)=0$	M1
	$z=-3$	A1
		(5)
		[5]
	Alternative	
	f(-3)=0 so $z=-3$ is also a root	M1A1
	$\left(z+3\right)\left(z^2+2z+5\right)=0$	
	$\left(z-\left(2i-1\right)\right)\left(z-\left(-2i-1\right)\right)=0$	M1A1
	$z=-2i-1$ is also a root	B1
	Notes	
	First M for expanding their $(z-\alpha)(z-\beta)$	
	Second M for inspection or long division.	

Question Number	Scheme	Marks
3(a)	$z_1 = \frac{1}{2} + i\frac{\sqrt{3}}{2}$ $r = \sqrt{\frac{1}{4}+\frac{3}{4}} = 1$, $\tan\theta = \sqrt{3}$ so $\theta = \dfrac{\pi}{3}$, both r values	M1A1
	$z_2 = -\sqrt{3} + i$ $r = \sqrt{3+1} = 2$, $\tan\theta = \dfrac{-1}{\sqrt{3}}$ so $\theta = \dfrac{5\pi}{6}$	M1A1 **(4)**
3(b)	$\lvert z_1 z_2 \rvert = \lvert z_1 \rvert \lvert z_2 \rvert = 2$	M1A1 **(2)**
3(c)		M1 A1ft **(2)** **[8]**
	Notes	
(a)	First M for use of Pythagoras, A1 for $r = 1$ and 2 Second M for use of $\tan$ or $\tan^{-1}$, A1 for $\theta = \dfrac{\pi}{3}$ and $\dfrac{5\pi}{6}$	
(b)	M for their $r_1 r_2$	
(c)	M for either of their numbers plotted correctly on a single diagram. A for both their numbers plotted correctly on a single diagram	

Question Number	Scheme	Marks
4(a)	$xy = 3$ or $y = \frac{3}{x}$	
	$x\frac{dy}{dx} + y = 0$	
	$\frac{dy}{dx} = \frac{-y}{x}$ or $\frac{dy}{dx} = -\frac{3}{x^2}$	M1A1
	Gradient of normal is $\frac{x}{y}$ or $\frac{x^2}{3}$	M1
	$y - 3 = \frac{1}{3}(x-1)$	M1
	$y = \frac{1}{3}x + \frac{8}{3}$	A1
		(5)
4(b)	At R, $y = \frac{3}{x}$	M1
	$\frac{9}{x} - x = 8$	
	$x^2 + 8x - 9 = 0$	A1
	$(x+9)(x-1) = 0$	M1
	$x = -9$, $y = -\frac{1}{3}$	A1,A1
		(5) **[10]**
	Notes	
(a)	First M: Use of the product rule: The sum of two terms including dy/dx, one of which is correct or $\frac{dy}{dx} = k\,x^{-2}$	
	First A for correct derivative $-3x^{-2}$ or $-\frac{y}{x}$	
	Second M for use of Perpendicular gradient rule $m_N m_T = -1$	
	Third M for $y - 3 =$ their $m_N\left(x-1\right)$ or $y = mx + c$ with their m_N and $(1,3)$ in an attempt to find 'c'.	
(b)	First M for substituting $y = \frac{3}{x}$ in their normal.	
	First A for correct 3 term quadratic	
	Second M for attempt to solve their 3 term quadratic	

Question Number	Scheme	Marks
5	$\mathrm{f}(1)=3^2+7=16=8\times 2$ True for $n=1$ Assume true for $n=k$, $\mathrm{f}(k)=3^{2k}+7=8p$ where p is a positive integer When $n=k+1$ $\mathrm{f}(k+1)-\mathrm{f}(k)=3^{2(k+1)}+7-\left(3^{2k}+7\right)$ $=9\times 3^{2k}+7-3^{2k}-7$ $=8\times 3^{2k}$ $\mathrm{f}(k+1)=8(3^{2k}+p)=8q$ where q is a positive integer <u>True for $n=k+1$</u> <u>True for $n=1$</u>, if <u>true for $n=k$</u> then true for $n=k+1$ So <u>$3^{2n}+7$ divisible by 8 for all n</u> by Induction.	B1 M1 dM1 A1 dM1 A1cso **(6)** **[6]**
	Notes B for $\mathrm{f}(1)=3^2+7=16$ seen First M for substituting into $\mathrm{f}(k+1)-\mathrm{f}(k)$ or showing $\mathrm{f}(k+1)=9\times 3^{2k}+7$ Second M for using $\mathrm{f}(k+1)-\mathrm{f}(k)$ or equivalent First A for $\mathrm{f}(k+1)=\mathrm{f}(k)+8\times 3^{2k}$ or equivalent. Third M for showing divisible by 8. Accept '$\mathrm{f}(k)$ divisible by 8 and 8×3^{2k} divisible by 8'. Second A for conclusion with all 4 underlined elements that can be seen anywhere in the solution	

Question Number	Scheme	Marks
6(a)	$y^2 = 4x$	
	$2y\dfrac{dy}{dx} = 4$	M1A1
	At P, $\dfrac{dy}{dx} = \dfrac{1}{p}$	A1
	$y - 2p = \dfrac{1}{p}\left(x - p^2\right)$	M1A1
		(5)
6(b)(i)	At (-1,2)	
	$2 - 2p = \dfrac{1}{p}\left(-1 - p^2\right)$	M1
	$p^2 - 2p - 1 = 0$	A1
	$p = 1 \pm \sqrt{2}$	M1
	$p = 1 + \sqrt{2}$, $q = 1 - \sqrt{2}$	A1
		(4)
6(b)(ii)	$PR^2 = 32 + 16\sqrt{2}$, $QR^2 = 32 - 16\sqrt{2}$	M1A1
	Area of $PQR = \dfrac{1}{2}PR.QR = 8\sqrt{2}$	M1A1 **(4)**
		[13]
	Notes	
(a)	First M for $x^{\frac{1}{2}} \to x^{-\frac{1}{2}}$ or $ky\dfrac{dy}{dx} = c$ or $\dfrac{dy}{dt} \times \dfrac{1}{\frac{dx}{dt}}$ **;** can be a function of *p or t*.	
	First A for accurate differentiation	
	Second M applies $y - 2p =$ their $m\left(x - p^2\right)$ or $y = \left(\text{their } m\right)x + c$ using $x = p^2$ *and* $y = 2p$ in an attempt to find c. **Their *m* must be a function of *p* from calculus.**	
(b)i	First M substitute coordinates of the point R into their tangent	
	Second M for solving 3 term quadratic	
	Second A for $1 \pm \sqrt{2}$ seen	
(b)ii	First M for attempt to find distance between their P and R or Q and R using formula or sketch and Pythagoras.	
	Second M for using $\dfrac{1}{2}bh$ on their PQR	
	Second A accept awrt 11.3	

Question Number	Scheme	Marks
7(a)	$\sum_{r=1}^{n} r^2(r-1) = \sum_{r=1}^{n} r^3 - \sum_{r=1}^{n} r^2$	M1
	$= \dfrac{n^2(n+1)^2}{4} - \dfrac{n(n+1)(2n+1)}{6}$	A1
	$= \dfrac{n(n+1)}{12}\left(3n\left(n+1\right)-2\left(2n+1\right)\right)$	M1
		A1
	$= \dfrac{n(n+1)(3n^2-n-2)}{12}$	
	$= \dfrac{n(n+1)(3n+2)(n-1)}{12}$	A1cso
		(5)
7(b)	$\sum_{r=10}^{r=50} r^2(r-1) = \sum_{r=1}^{50} r^2(r-1) - \sum_{r=1}^{r=9} r^2(r-1)$	M1
	$= \frac{1}{12}\left(50\times 51\times 152\times 49\right) - \frac{1}{12}\left(9\times 10\times 29\times 8\right)$	A1
	$= 1582700 - 1740 = 1580960$	A1
		(3)
		[8]
	Notes	
(a)	First M for expanding brackets First A for correct expressions for $\sum r^3$ and $\sum r^2$ Second M for factorising by $n(n+1)$ Second A for $(3n^2-n-2)$ or equivalent factor	
(b)	First M for f(49 or 50) – f(9 or 10) and attempt to use part (a).	

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>8(a)</td><td>$(\mathrm{f}(1) =) -4 (< 0)$ -4
$(\mathrm{f}(2) =) 1 (> 0)$ 1
Changes sign so root (in [1,2])</td><td>B1
B1
B1
(3)</td></tr>
<tr><td>8(b)</td><td>
<table>
<tr><th>a</th><th>$\mathrm{f}(a)$</th><th>b</th><th>$\mathrm{f}(b)$</th><th>$\frac{a+b}{2}$</th><th>$\mathrm{f}\left(\frac{a+b}{2}\right)$</th></tr>
<tr><td>1</td><td>-4</td><td>2</td><td>1</td><td>1.5</td><td>-2.625</td></tr>
<tr><td>1.5</td><td>-2.625</td><td>2</td><td>1</td><td>1.75</td><td>-1.140625</td></tr>
</table>

Interval is [1.75,2]</td><td>B1M1

A1
(3)</td></tr>
<tr><td>8(c)</td><td>$\mathrm{f}'(x) = 3x^2 - 2$

$x_1 = 1.8 - \frac{1.8^3 - 2\times 1.8 - 3}{3\times 1.8^2 - 2}$
$x_1 = 1.90$ to 3sf.</td><td>M1A1

M1A1

A1
(5)
[11]</td></tr>
<tr><td>
(b)

(c)</td><td>Notes
B for awrt -2.6
M for attempt to find $\mathrm{f}(1.75)$
A for $\mathrm{f}(1.75) =$ awrt -1.1 with $1.75 \leqslant \alpha \leqslant 2$ or $1.75 < \alpha < 2$
or $[1.75,\ 2]$ or $(1.75,\ 2)$.
First M for at least one of the two terms differentiated correctly.
First A for correct derivative
Second M for correct application of Newton-Raphson using their values.
Second A for f(1.8)= -0.768
Third A for 1.90 cao</td><td></td></tr>
</table>

Question Number	Scheme	Marks
9(a)	$\begin{pmatrix} 3 & 1 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} 10 & 1 \\ 1 & 5 \end{pmatrix}$	M1A1 **(2)**
(b)	$\det \mathbf{A} = -7 \neq 0$ so $\mathbf{A}$ is non-singular	M1A1 **(2)**
(c)	$\mathbf{A}^{-1} = -\frac{1}{7}\begin{pmatrix} -2 & -1 \\ -1 & 3 \end{pmatrix}$	M1A1 **(2)**
(d)	$-\frac{1}{7}\begin{pmatrix} -2 & -1 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} k-1 \\ 2-k \end{pmatrix} = -\frac{1}{7}\begin{pmatrix} -2(k-1)-1(2-k) \\ -1(k-1)+3(2-k) \end{pmatrix}$	M1
	$= \begin{pmatrix} \frac{1}{7}k \\ \frac{4}{7}k-1 \end{pmatrix}$	A1,A1
	(p lies on $y = 4x-1$)	**(3)**
		[9]
	Notes	
(d)	**Alt**	
	$\begin{pmatrix} 3 & 1 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} k-1 \\ 2-k \end{pmatrix}$ then multiply out and attempt to solve simultaneous equations for x or y in terms of k. M1 $x = \frac{1}{7}k$ A1 $y = \frac{4}{7}k-1$ A1	

Further copies of this publication are available from
Edexcel Publications, Adamsway, Mansfield, Notts, NG18 4FN

Telephone 01623 467467
Fax 01623 450481
Email publication.orders@edexcel.com

Order Code UA035965 Summer 2013

For more information on Edexcel qualifications, please visit our website
www.edexcel.com

Pearson Education Limited. Registered company number 872828
with its registered office at Edinburgh Gate, Harlow, Essex CM20 2JE

Ofqual

Llywodraeth Cynulliad Cymru
Welsh Assembly Government

cea
Rewarding Learning