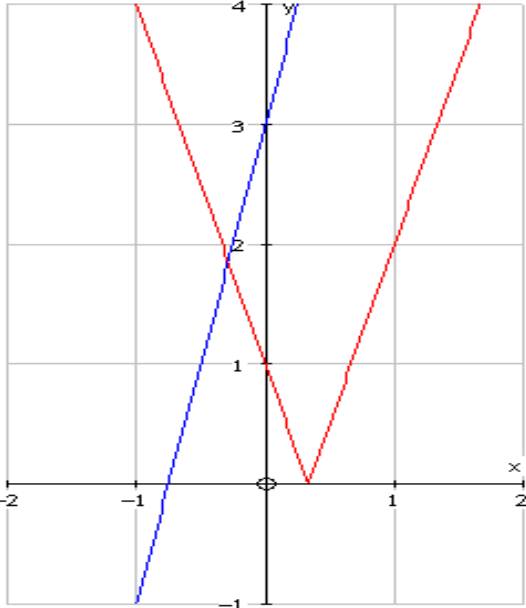


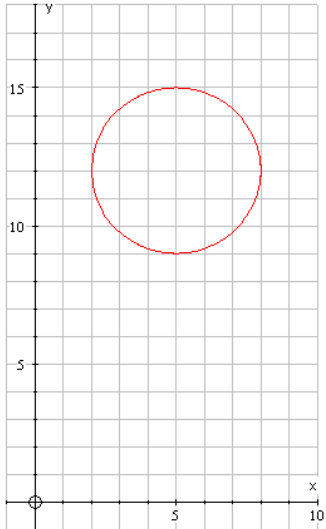
Further Pure Mathematics FP2 (6668)

Mock paper mark scheme

Question number	Scheme	Marks
<p>1. (a)</p>	 <p>Line correct</p> <p>V shape correct</p> <p>$\frac{1}{3}$ and $-\frac{3}{4}$</p> <p>(b) Point of intersection when $4x + 3 = 1 - 3x$, and so $x = -\frac{2}{7}$</p> <p>Solution is $x > -\frac{2}{7}$</p>	<p>B1</p> <p>B1</p> <p>B1 (3)</p> <p>M1 A1</p> <p>A1 (3)</p> <p>(6 marks)</p>
<p>2. (a)</p> <p>(b)</p>	$\frac{1}{2r+1} - \frac{1}{2r+3}$ $\sum = \frac{1}{3} - \frac{1}{5} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2n+1} - \frac{1}{2n+3}$ $= \frac{1}{3} - \frac{1}{2n+3} = \frac{2n+3-3}{3(2n+3)} = \frac{2n}{3(2n+3)} (*)$	<p>M1 A1 (2)</p> <p>M1 A1</p> <p>A1 cso (3)</p> <p>(5 marks)</p>

Question number	Scheme	Marks
3.	<p>(a) $\frac{dy}{dx} = \frac{5}{1+5x}, \quad \frac{d^2y}{dx^2} = -\frac{25}{(1+5x)^2}, \quad \frac{d^3y}{dx^3} = \frac{250}{(1+5x)^3}$</p> <p>(b) $\ln(1+5x) = 5x - \frac{25}{2}x^2 + \frac{125}{3}x^3 + \dots$</p>	<p>M1 A1, A1 A1 (4)</p> <p>M1 A1 A1 (3)</p> <p>(7 marks)</p>
4.	<p>$\frac{d^2y}{dx^2} + 1 + 1 = 4$ at $x = 0, \quad \therefore \frac{d^2y}{dx^2} = 2$</p> <p>Differentiate to give</p> <p>$\frac{d^3y}{dx^3} + \left[\left(\frac{dy}{dx}\right)^2 + y \frac{d^2y}{dx^2}\right] + 2y \frac{dy}{dx} = 3$</p> <p>At $x = 0, \quad \frac{d^3y}{dx^3} + [1^2 + 1 \times 2] + 2 = 3$ and $\frac{d^3y}{dx^3} = -2$</p> <p>$y = 1 + x + \frac{2x^2}{2} - \frac{2x^3}{6} + \dots$</p>	<p>B1</p> <p>M1 [M1 A1] A1</p> <p>B1</p> <p>M1 A1</p> <p>(8 marks)</p>
5.	<p>Area = $\frac{1}{2} \int_0^{\frac{\pi}{2}} (4 + 4 \sin 3\theta + \sin^2 3\theta) d\theta$</p> <p>$= \frac{1}{2} \left[4\theta - \frac{4 \cos 3\theta}{3} + \frac{\theta}{2} - \frac{\sin 6\theta}{12} \right]_0^{\frac{\pi}{2}}$</p> <p>$= \frac{1}{2} \left(2\pi + \frac{\pi}{4} \right) - \frac{1}{2} \left(-\frac{4}{3} \right)$</p> <p>$= \frac{9\pi}{8} + \frac{2}{3}$</p>	<p>M1</p> <p>M1 A1 M1 A1</p> <p>M1</p> <p>A1 (7)</p> <p>(7 marks)</p>

Question number	Scheme	Marks
6.	<p>(a) $i \sin 5\theta = \text{Im}(\cos \theta + i \sin \theta)^5$</p> $= i(5 \cos^4 \theta \sin \theta - 10 \cos^2 \theta \sin^3 \theta + \sin^5 \theta)$ $= i(5(1 - \sin^2 \theta)^2 \sin \theta - 10(1 - \sin^2 \theta) \sin^3 \theta + \sin^5 \theta)$ $\therefore \sin 5\theta = 16 \sin^5 \theta - 20 \sin^3 \theta + 5 \sin \theta$ <p>(5)</p> <p>(b) Put $5 \sin \theta = 16 \sin^5 \theta - 20 \sin^3 \theta + 5 \sin \theta$</p> $\therefore 16 \sin^5 \theta - 20 \sin^3 \theta = 0$ $\therefore \sin \theta = 0 \text{ or } \sin \theta = \pm \sqrt{\frac{5}{4}} \text{ (no solution as } \sin \theta > 1)$ <p>So only solutions are $\theta = n\pi$.</p>	<p>M1</p> <p>M1 A1</p> <p>M1</p> <p>A1 A1</p> <p>A1 (4)</p> <p>(9 marks)</p>
7.	<p>(a) Integrating factor is $e^{-\int 0.1 dt} = e^{-0.1t}$</p> <p>Use to obtain $P e^{-0.1t} = \int 0.05 t e^{-0.1t} dt$</p> $= \frac{-0.05 t e^{-0.1t}}{0.1} + \int \frac{0.05 e^{-0.1t}}{0.1} dt$ $= -0.5 t e^{-0.1t} - 5 e^{-0.1t} + c$ $\therefore P = -\frac{1}{2} t - 5 + c e^{0.1t}$ <p>But at $t = 0$, $P = 10000$</p> <p>So $c = 10005$ and $\therefore P = -\frac{1}{2} t - 5 + 10005 e^{0.1t}$</p> <p>(b) When $t = 6$, $P = 18222 < 20000$</p> <p>When $t = 7$, $P = 20139 > 20000$</p> <p>So P reaches 20 000 during the seventh year..</p>	<p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1 A1 (7)</p> <p>M1</p> <p>A1 (2)</p> <p>(9 marks)</p>

Question number	Scheme	Marks
8.	<p>(a) </p> <p>Locus is a circle</p> <p>Centre is at (5, 12)</p> <p>Radius is 3</p> <p>(b) Finds distance from centre to origin is 13</p> <p>Maximum modulus is $13 + 3 = 16$</p> <p>Minimum modulus is $13 - 3 = 10$</p> <p>(c) Finds $\arctan \frac{12}{5}$</p> <p>Uses $\arctan \frac{12}{5} \pm \arcsin \frac{3}{13}$</p> <p>Obtains 0.94 and 1.41 radians</p>	<p>B1</p> <p>B1</p> <p>B1 (3)</p> <p>M1</p> <p>M1 A1</p> <p>A1(4)</p> <p>M1</p> <p>M1</p> <p>A1 A1 (4)</p> <p>(11 marks)</p>

Question number	Scheme	Marks
9.	(a) $V = \lambda t \sin 8t, \quad \frac{dV}{dt} = \lambda \sin 8t + 8\lambda t \cos 8t$	M1, A1
	Substitute to give $\frac{d^2V}{dt^2} = 16\lambda \cos 8t + 64\lambda t \sin 8t$	A1
	$16\lambda \cos 8t = \cos 8t, \quad \text{and } \therefore \lambda = \frac{1}{16}$	M1, A1 (5)
	(b) Auxiliary equation is $m^2 + 64 = 0$ and so $m = \pm 8i$	B1
	Complementary function is $A \cos 8t + B \sin 8t$ General solution is $A \cos 8t + B \sin 8t + \frac{1}{16} t \sin 8t$	M1 A1 B1 (4)
	(c) $V = 0$, when $t = 0$ implies $A = 0$	
	$8B \cos 8t + \frac{1}{16} \sin 8t + \frac{1}{2} t \cos 8t = 0$ when $t = 0$ So $8B = 0$ and $V = \frac{1}{16} t \sin 8t$ is particular solution.	(3)
	(d) As t becomes large the amplitude of the oscillations of V become large also. As $t \rightarrow \infty, \quad V \rightarrow \infty$ also.	B1 (1) (13 marks)