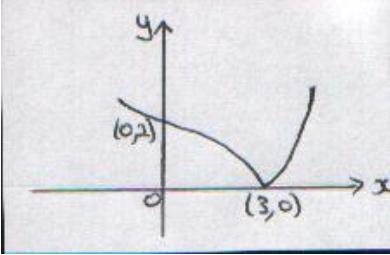
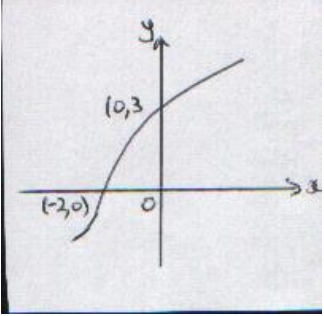
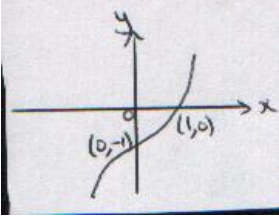


**June 2006**  
**6665 Core Mathematics C3**  
**Mark Scheme**

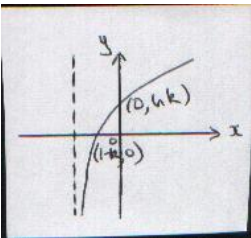
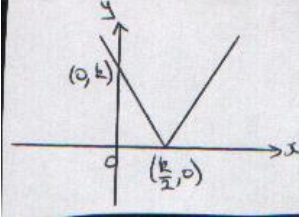
Question number	Scheme	Marks
1.	<p>(a) <math>\frac{(3x+2)(x-1)}{(x+1)(x-1)}, = \frac{3x+2}{x+1}</math></p> <p><b>Notes</b>  M1 attempt to factorise numerator, <i>usual rules</i>  B1 factorising denominator seen anywhere in (a),  A1 given answer  If factorisation of denom. not seen, correct answer implies B1</p> <p>(b) Expressing over common denominator</p> $\frac{3x+2}{x+1} - \frac{1}{x(x+1)} = \frac{x(3x+2)-1}{x(x+1)}$ <p>[Or “Otherwise” : <math>\frac{(3x^2 - x - 2)x - (x - 1)}{x(x^2 - 1)}</math> ]</p> <p>Multiplying out numerator and attempt to factorise  <math>[3x^2 + 2x - 1 \equiv (3x - 1)(x + 1)]</math></p> <p>Answer: <math>\frac{3x-1}{x}</math></p>	<p>M1B1, A1 (3)</p> <p>M1</p> <p>M1</p> <p>A1 (3)</p> <p><b>Total 6 marks</b></p>
2.	<p>(a) <math>\frac{dy}{dx} = 3e^{3x} + \frac{1}{x}</math></p> <p><b>Notes</b>  B1 <math>3e^{3x}</math>  M1 : <math>\frac{a}{bx}</math>      A1: <math>3e^{3x} + \frac{1}{x}</math></p>	<p>B1M1A1 (3)</p>
	<p>(b) <math>(5+x^2)^{\frac{1}{2}}</math></p> $\frac{3}{2}(5+x^2)^{\frac{1}{2}} \cdot 2x = 3x(5+x^2)^{\frac{1}{2}} \quad \text{M1 for}$ <p><math>kx(5+x^2)^m</math></p>	<p>B1</p> <p>M1 A1 (3)</p> <p><b>Total 6 marks</b></p>

Question Number	Scheme	Marks
3. (a)	 <p data-bbox="847 383 1214 416">Mod graph, reflect for <math>y &lt; 0</math></p> <p data-bbox="847 472 1190 539"><math>(0, 2), (3, 0)</math> or marked on axes</p> <p data-bbox="847 607 1238 640">Correct shape, including cusp</p>	<p data-bbox="1270 383 1310 416">M1</p> <p data-bbox="1270 495 1310 528">A1</p> <p data-bbox="1270 607 1310 640">A1</p> <p data-bbox="1414 607 1461 640">(3)</p>
3. (b)	 <p data-bbox="847 741 1222 775">Attempt at reflection in <math>y = x</math></p> <p data-bbox="847 909 1078 943">Curvature correct</p> <p data-bbox="847 1077 1166 1111"><math>-2, 0), (0, 3)</math> or equiv.</p>	<p data-bbox="1270 741 1310 775">M1</p> <p data-bbox="1270 909 1310 943">A1</p> <p data-bbox="1270 1077 1310 1111">B1</p> <p data-bbox="1414 1077 1461 1111">(3)</p>
3. (c)	 <p data-bbox="847 1245 1126 1279">Attempt at 'stretches'</p> <p data-bbox="847 1424 1054 1458"><math>(0, -1)</math> or equiv.</p> <p data-bbox="847 1603 935 1637"><math>(1, 0)</math></p>	<p data-bbox="1270 1245 1310 1279">M1</p> <p data-bbox="1270 1413 1310 1447">B1</p> <p data-bbox="1270 1603 1310 1637">B1</p> <p data-bbox="1414 1603 1461 1637">(3)</p>
		<b>Total 9 marks</b>

Question Number	Scheme	Marks
<p><b>4.</b></p> <p>(a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p>	<p>425 °C</p> <p><math>300 = 400e^{-0.05t} + 25 \Rightarrow 400e^{-0.05t} = 275</math></p> <p>sub. <math>T = 300</math> and attempt to rearrange to <math>e^{-0.05t} = a</math>, where <math>a \in \mathbb{Q}</math></p> <p><math>e^{-0.05t} = \frac{275}{400}</math></p> <p>M1 correct application of logs</p> <p><math>t = 7.49</math></p> <p><math>\frac{dT}{dt} = -20 e^{-0.05t}</math> (M1 for <math>ke^{-0.05t}</math>)</p> <p>At <math>t = 50</math>, rate of decrease = <math>(\pm) 1.64</math> °C/min</p> <p><math>T &gt; 25</math>, (since <math>e^{-0.05t} \rightarrow 0</math> as <math>t \rightarrow \infty</math>)</p>	<p>B1 (1)</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 (4)</p> <p>M1 A1</p> <p>A1 (3)</p> <p>B1 (1)</p> <p><b>Total 9 marks</b></p>

Question Number	Scheme	Marks
5. (a)	<p>Using product rule: <math>\frac{dy}{dx} = 2 \tan 2x + 2(2x - 1) \sec^2 2x</math></p> <p>Use of “<math>\tan 2x = \frac{\sin 2x}{\cos 2x}</math>” and “<math>\sec 2x = \frac{1}{\cos 2x}</math>”</p> <p><math>[= 2 \frac{\sin 2x}{\cos 2x} + 2(2x - 1) \frac{1}{\cos^2 2x} ]</math></p> <p>Setting <math>\frac{dy}{dx} = 0</math> and multiplying through to eliminate fractions</p> <p><math>[ \Rightarrow 2 \sin 2x \cos 2x + 2(2x - 1) = 0 ]</math></p> <p>Completion: producing <math>4k + \sin 4k - 2 = 0</math> with no wrong working seen and at least previous line seen. AG</p>	<p>M1 A1 A1</p> <p>M1</p> <p>M1</p> <p>A1* (6)</p>
(b)	<p><math>x_1 = 0.2670, x_2 = 0.2809, x_3 = 0.2746, x_4 = 0.2774,</math></p> <p><b>Note:</b> M1 for first correct application, first A1 for two correct, second A1 for all four correct</p> <p>Max -1 deduction, if ALL correct to &gt; 4 d.p. M1 A0 A1</p> <p>SC: degree mode: M1 <math>x_1 = 0.4948</math> , A1 for <math>x_2 = 0.4914</math>, then A0; max 2</p>	<p>M1 A1 A1 (3)</p>
(c)	<p>Choose suitable interval for <math>k</math>: e.g. <math>[0.2765, 0.2775]</math> and evaluate <math>f(x)</math> at these values</p> <p>Show that <math>4k + \sin 4k - 2</math> changes sign and deduction</p> <p><math>[f(0.2765) = -0.000087.., f(0.2775) = +0.0057]</math></p> <p><b>Note:</b></p> <p>Continued iteration: (no marks in degree mode)</p> <p>Some evidence of further iterations leading to 0.2765 or better</p> <p>M1;</p> <p>Deduction A1</p>	<p>M1</p> <p>A1 (2)</p> <p>(11 marks)</p>

Question Number	Scheme	Marks
6. (a)	Dividing $\sin^2 \theta + \cos^2 \theta \equiv 1$ by $\sin^2 \theta$ to give $\frac{\sin^2 \theta}{\sin^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta} \equiv \frac{1}{\sin^2 \theta}$ Completion: $1 + \cot^2 \theta \equiv \operatorname{cosec}^2 \theta \Rightarrow \operatorname{cosec}^2 \theta - \cot^2 \theta \equiv 1$ AG	M1 A1* (2)
(b)	$\operatorname{cosec}^4 \theta - \cot^4 \theta \equiv (\operatorname{cosec}^2 \theta - \cot^2 \theta)(\operatorname{cosec}^2 \theta + \cot^2 \theta)$ $\equiv (\operatorname{cosec}^2 \theta + \cot^2 \theta) \quad \text{using (a)} \quad \text{AG}$ <p><b>Notes:</b>            (i) Using LHS = <math>(1 + \cot^2 \theta)^2 - \cot^4 \theta</math>, using (a) &amp; elim. <math>\cot^4 \theta</math>            M1,            conclusion {using (a) again} A1*            (ii) Conversion to sines and cosines: needs  <math display="block">\frac{(1 - \cos^2 \theta)(1 + \cos^2 \theta)}{\sin^4 \theta}</math> for M1</p>	M1 A1* (2)
(c)	Using (b) to form $\operatorname{cosec}^2 \theta + \cot^2 \theta \equiv 2 - \cot \theta$ Forming quadratic in $\cot \theta$ $\Rightarrow 1 + \cot^2 \theta + \cot^2 \theta \equiv 2 - \cot \theta \quad \{\text{using (a)}\}$ $2\cot^2 \theta + \cot \theta - 1 = 0$ Solving: $(2\cot \theta - 1)(\cot \theta + 1) = 0$ to $\cot \theta =$ $\left(\cot \theta = \frac{1}{2}\right) \quad \text{or} \quad \cot \theta = -1$ $\theta = 135^\circ \quad (\text{or correct value(s) for candidate dep. on 3Ms})$ <p><b>Note:</b> Ignore solutions outside range            Extra “solutions” in range loses A1√, but candidate may possibly have more than one “correct” solution.</p>	M1 M1 A1 M1 A1 A1√ (6)  <b>(10 marks)</b>

Question Number	Scheme	Marks
7. (a)	 <p>Log graph: Shape</p> <p>Intersection with -ve <math>x</math>-axis</p> <p><math>(0, \ln k), (1 - k, 0)</math></p>	B1 dB1 B1
	 <p>Mod graph :V shape, vertex on +ve <math>x</math>-axis</p> <p><math>(0, k)</math> and <math>(\frac{k}{2}, 0)</math></p>	B1 B1 (5)
(b)	$f(x) \in \mathbb{R}, -\infty < f(x) < \infty, -\infty < y < \infty$	B1 (1)
(c)	$f\left(\frac{k}{4}\right) = \ln\left\{k + \left \frac{2k}{4} - k\right \right\}$ or $f\left(\left -\frac{k}{2}\right \right)$ $= \ln\left(\frac{3k}{2}\right)$	M1 A1 (2)
(d)	$\frac{dy}{dx} = \frac{1}{x+k}$ Equating (with $x = 3$ ) to grad. of line; $\frac{1}{3+k} = \frac{2}{9}$ $k = 1\frac{1}{2}$	B1 M1; A1 A1√ (4) <b>(12 marks)</b>

Question Number	Scheme	Marks
<p>8. (a)</p> <p>(b)(i)</p> <p>(b)(ii)</p>	<p>Method for finding <math>\sin A</math></p> $\sin A = -\frac{\sqrt{7}}{4}$ <p><b>Note:</b> First A1 for <math>\frac{\sqrt{7}}{4}</math>, exact. Second A1 for sign (even if dec. answer given)</p> <p>Use of <math>\sin 2A \equiv 2 \sin A \cos A</math></p> $\sin 2A = -\frac{3\sqrt{7}}{8} \text{ or equivalent exact}$ <p><b>Note:</b> <math>\pm</math> f.t. Requires exact value, dependent on 2nd M</p> $\cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right) \equiv \cos 2x \cos \frac{\pi}{3} - \sin 2x \sin \frac{\pi}{3} +$ $\cos 2x \cos \frac{\pi}{3} + \sin 2x \sin \frac{\pi}{3}$ $\equiv 2 \cos 2x \cos \frac{\pi}{3}$ <p>[This can be just written down (using factor formulae) for M1A1]</p> $\equiv \cos 2x \quad \text{AG}$ <p><b>Note:</b> M1A1 earned, if <math>\equiv 2 \cos 2x \cos \frac{\pi}{3}</math> just written down, using factor theorem Final A1* requires some working after first result.</p> $\frac{dy}{dx} = 6 \sin x \cos x - 2 \sin 2x$ <p>or <math>6 \sin x \cos x - 2 \sin\left(2x + \frac{\pi}{3}\right) - 2 \sin\left(2x - \frac{\pi}{3}\right)</math></p> $= 3 \sin 2x - 2 \sin 2x$ $= \sin 2x \quad \text{AG}$ <p><b>Note:</b> First B1 for <math>6 \sin x \cos x</math>; second B1 for remaining term(s)</p>	<p>M1</p> <p>A1 A1</p> <p>M1</p> <p>A1√ (5)</p> <p>M1</p> <p>A1</p> <p>A1* (3)</p> <p>B1 B1</p> <p>M1</p> <p>A1* (4)</p> <p>(12 marks)</p>